Example 17

Given the function $y = 3\sin t$, for $t > 0$:

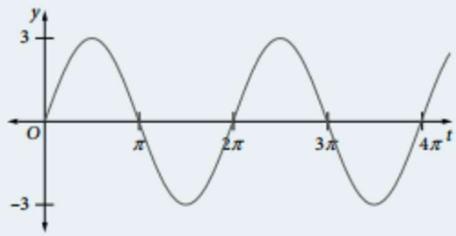
(a) Sketch the graph of this function for $0 \le t \le 4\pi$.

(b) Find the greatest and least values of the function and where they occur for $0 \le t \le 4\pi$.

(c) Describe the behaviour of the curve if y is the distance in metres to the right of a fixed point after a time t hours.

Solution

(a) $y = 3\sin t$, for $0 \le t \le 4\pi$:



(b) $\frac{dy}{dt} = 3\cos t$. For stationary points $\frac{dy}{dt} = 0$, so $\cos t = 0$.

For $0 \le t \le 4\pi$: $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\frac{d^2y}{dt^2} = -3\sin t$: at $t = \frac{\pi}{2}, \frac{5\pi}{2}$ we have $\frac{d^2y}{dt^2} < 0$ $\therefore$ maximum turning point

at $t = \frac{3\pi}{2}, \frac{7\pi}{2}$ we have $\frac{d^2y}{dt^2} > 0$ $\therefore$ minimum turning point

At $t = \frac{\pi}{2}, \frac{5\pi}{2}$, $y = 3$. The greatest value of the function is 3 and occurs at $t = \frac{\pi}{2}, \frac{5\pi}{2}$.

At $t = \frac{3\pi}{2}, \frac{7\pi}{2}$, $y = -3$. The least value of the function is -3 and occurs at $t = \frac{3\pi}{2}, \frac{7\pi}{2}$.

(c) As t increases, y increases and then decreases in a repeating pattern. The function is periodic with a period of 2π hours (i.e. about 6 h 17 min). Its greatest distance from the fixed point is 3 metres in either direction. After moving 3 metres to the right, the object moves back through its starting point until it reaches a point 3 metres to the left; it then starts moving back to the right again until the pattern repeats.

The kind of wave-like repeating movement described in Example 17 is called **simple harmonic motion**. It is a type of motion that can be described using sine and cosine functions. It occurs naturally in the motion of pendulums and masses on springs, and it can also be used to model ocean tides and other wave-like movements.