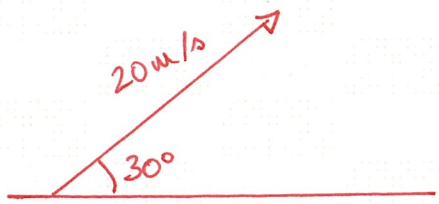
APPLICATIONS TO PHYSICAL SITUATIONS (from CAMBRIDGE)

In this exercise take $g = 9.8 \text{ m/s}^2$.

1 A ball is thrown at an angle of 30° to the horizontal with an initial speed of 20 m/s. Find the initial horizontal and vertical components of the velocity of the ball.



$V_x = 20 \cos 30 = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m s}^{-1}$

$V_y = 20 \sin 30 = 20 \times \frac{1}{2} = 10 \text{ m s}^{-1}$

2 A particle has initial position vector $(4\underset{\sim}{i} + 5\underset{\sim}{j})$ metres. It moves with a constant velocity of $(3\underset{\sim}{i} - 2\underset{\sim}{j})$ m/s. Find its position vector after 7 seconds.

$x(7) = 4 + 3 \times 7 = 4 + 21 = 25$

$y(7) = 5 + (-2) \times 7 = 5 - 14 = -9$

So the particle position vector at $t = 7$ is $25\underset{\sim}{i} - 9\underset{\sim}{j}$

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3 Find the magnitude of the resultant of the forces $(2\underset{\sim}{i} - 3\underset{\sim}{j})$ N, $(4\underset{\sim}{i} + \underset{\sim}{j})$ N and $(-3\underset{\sim}{i} + 3\underset{\sim}{j})$ N.

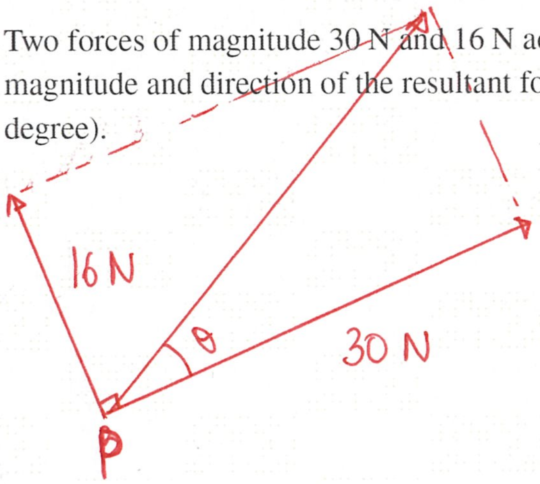
$$\vec{F_1} + \vec{F_2} + \vec{F_3} = (2\vec{i} - 3\vec{j}) + (4\vec{i} + \vec{j}) + (-3\vec{i} + 3\vec{j})$$

$$= (2+4-3)\vec{i} + (-3+1+3)\vec{j}$$

$$= 3\vec{i} + \vec{j}$$

$$|\vec{F_1} + \vec{F_2} + \vec{F_3}| = |3\vec{i} + \vec{j}| = \sqrt{3^2 + 1^2} = \sqrt{10} \text{ N}$$

4 Two forces of magnitude 30 N and 16 N act away from a point P and are perpendicular. Find the magnitude and direction of the resultant force (measured from the 30 N force correct to the nearest degree).

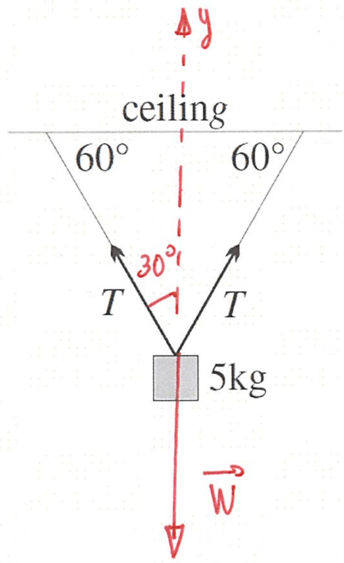


The magnitude of the resulting force is $\sqrt{30^2 + 16^2} = 34$ N

θ is such that $\tan\theta = \frac{16}{30} = \frac{8}{15}$

so $\theta \simeq 28°$

6 In the diagram, an object of mass 5 kg is suspended from a horizontal ceiling by two strings of equal length. Each string makes an angle of 60° with the ceiling. Calculate, correct to 3 significant figures, the equal tensions in the two strings.



By projecting all forces on the y-axis, we obtain: $T\cos 30 + T\cos 30 = |\vec{W}|$

$\Leftrightarrow \quad 2T\cos 30 = mg$

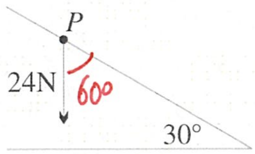
$\Leftrightarrow \quad 2T \times \frac{\sqrt{3}}{2} = 5 \times 9.8$

$\Leftrightarrow \quad T = \frac{5 \times 9.8}{\sqrt{3}} \simeq 28.3$ Newtons

7 The diagram shows an object of weight 24 N at rest at P on an inclined plane. Find the component of the weight:

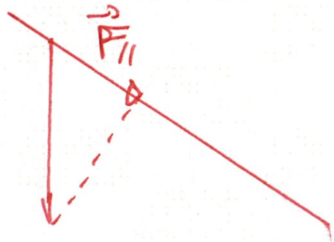
a down the plane,

b perpendicular to the plane.



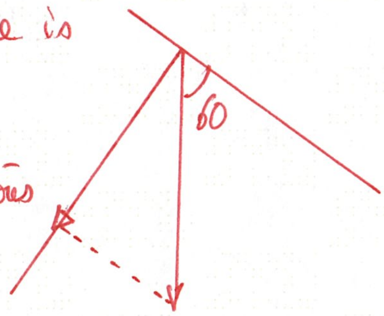
a) The projection on the plane gives.

$$|\vec{F}_{/\!/}| = 24 \cos 60 = 24 \times \frac{1}{2} = 12 \text{ Newtons}$$

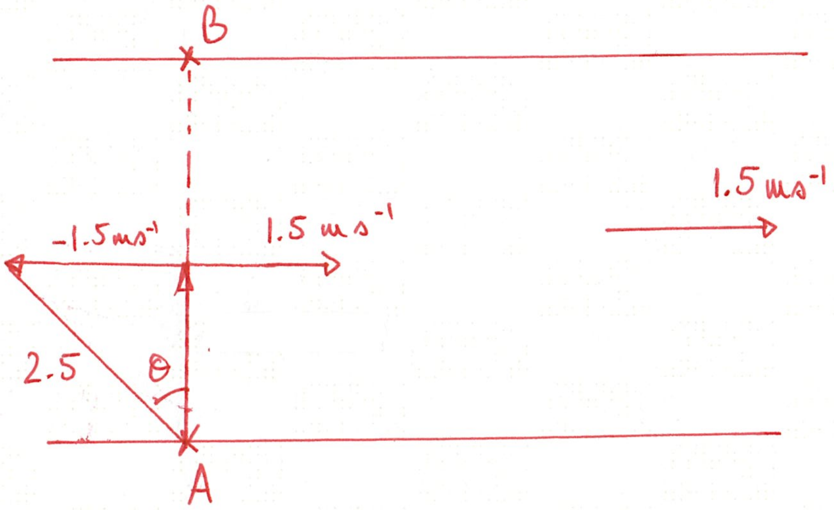


b) The projection perpendicular to the plane is

$$|\vec{F}_{\perp}| = 24 \sin 60 = 24 \times \frac{\sqrt{3}}{2} = 12\sqrt{3} \text{ Newtons}$$



10 A river is flowing at a speed of 1.5 m/s. Sam wants to row from point A on one bank to point B on the other bank directly opposite A. He intends to maintain a constant speed of 2.5 m/s. In what direction, correct to the nearest degree, should Sam row? Give your answer as an angle of inclination to the line AB.

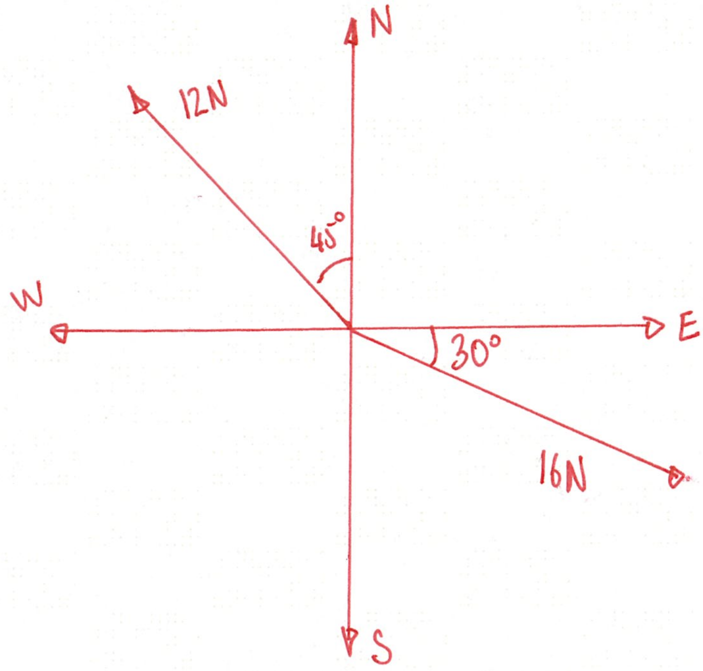


$$\sin\theta = \frac{1.5}{2.5} = \frac{3}{5}$$

$$\therefore \quad \theta = \sin^{-1}\frac{3}{5} \approx 37^\circ$$

APPLICATIONS TO PHYSICAL SITUATIONS (from CAMBRIDGE)

11 Two dogs Brutus and Nitro are simultaneously tugging on a bone. Brutus is pulling with a force of 12 N in a direction 45° west of north, while Nitro is pulling with a force of 16 N in a direction 30° south of east. Calculate, correct to two significant figures, the magnitude and direction of the resultant force.



$$\overrightarrow{\text{Resulting force}} = \left[12\cos 135\,\vec{i} + 12\sin 135\,\vec{j}\right] + \left[16\cos(-30)\,\vec{i} + 16\sin(-30)\,\vec{j}\right]$$

$$\text{———} = \left[12 \times \left(-\frac{\sqrt{2}}{2}\right) + 16 \times \frac{\sqrt{3}}{2}\right]\vec{i} + \left[12 \times \frac{\sqrt{2}}{2} - 16 \times \frac{1}{2}\right]\vec{j}$$

$$\text{———} = \left[-6\sqrt{2} + 8\sqrt{3}\right]\vec{i} + \left[6\sqrt{2} - 8\right]\vec{j}$$

$$|\overrightarrow{\text{Force}}| = \sqrt{(-6\sqrt{2} + 8\sqrt{3})^2 + (6\sqrt{2} - 8)^2} = 5.39 \text{ Newtons}$$

$$\tan\theta = \frac{6\sqrt{2} - 8}{-6\sqrt{2} + 8\sqrt{3}} \qquad \therefore \theta = \tan^{-1}\left[\frac{6\sqrt{2} - 8}{-6\sqrt{2} + 8\sqrt{3}}\right] \simeq 5.2°$$

12 Three forces act on an object of mass 5 kg. These forces are represented by the vectors $9\underset{\sim}{i} - 2\underset{\sim}{j}$, $-3\underset{\sim}{i} + 10\underset{\sim}{j}$ and $18\underset{\sim}{i} - \underset{\sim}{j}$. Calculate the magnitude and direction of the acceleration of the object.

$$m\vec{a} = \sum \vec{F} = [9\vec{i} - 2\vec{j}] + [-3\vec{i} + 10\vec{j}] + [18\vec{i} - \vec{j}]$$

$$\therefore \vec{a} = \frac{1}{m}\left[(9 - 3 + 18)\vec{i} + (-2 + 10 - 1)\vec{j}\right]$$

$$\therefore \vec{a} = \frac{1}{5}\left[24\vec{i} + 7\vec{j}\right]$$

$$|\vec{a}| = \sqrt{\left(\frac{24}{5}\right)^2 + \left(\frac{7}{5}\right)^2} = 5$$

$$\tan\theta = \frac{7/5}{24/5} = \frac{7}{24} \qquad \therefore \theta = \tan^{-1}\left(\frac{7}{24}\right) \simeq 16^\circ$$

13 The position of a plane flying horizontally in a straight line at a constant speed is plotted on a radar screen. One unit on the screen represents 1 km in the air. At 12 noon the position vector of the plane is $40\underset{\sim}{i} + 16\underset{\sim}{j}$. Five minutes later its position vector is $33\underset{\sim}{i} + 40\underset{\sim}{j}$. Find:

a the position vector of the plane at 12:15 pm,

b the velocity of the plane as a vector in km/h.

a) At 12.00 $\quad r(\cancel{12}) = 40\vec{i} + 16\vec{j}$

At 12.05 $\quad r(12.05) = 33\vec{i} + 40\vec{j}$

So in 5', the change is $-7\vec{i} + 24\vec{j}$

∴ in 10' ———— would be $-14\vec{i} + 48\vec{j}$.

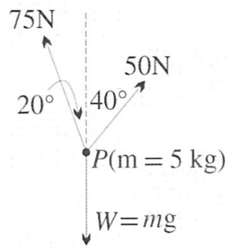
Hence, at 12.15, $\vec{r}(12.15) = [33-14]\vec{i} + [40+48]\vec{j}$

$\vec{r}(12.15) = 19\vec{i} + 88\vec{j}$

b) in 5 min, the change is $-7\vec{i} + 24\vec{j}$

Multiplying by 12, ———— $-84\vec{i} + 288\vec{j}$

14 The diagram shows an object of mass 5 kg being raised by forces of magnitude 75 N and 50 N.



a Find the weight of the object.

b Find, correct to the nearest newton, the magnitude of the resultant of the three forces acting on the object.

c Find, correct to the nearest degree, the angle this resultant makes with the upward vertical direction.

a) $W = mg = 5 \times 9.8 = 49\,N$

b) Summing the 3 forces on the vertical, the resulting force must be equal to $75 \cos 20 + 50 \cos 40 - 49$

$= 60$ Newtons approx.

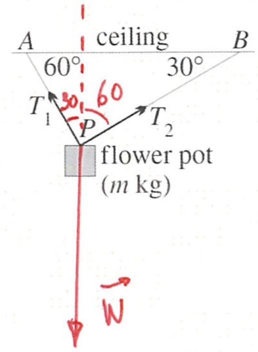
c) Summing the 3 forces on the horizontal results in:

$$\vec{F}_{horizontal} = \left[75 \cos(90+20) + 50 \cos 50\right]$$

$$= 6.5\vec{i}$$

$\therefore \tan\theta = \dfrac{6.5}{60}$ $\therefore \theta \simeq \tan^{-1}\dfrac{6.5}{60} \simeq 6^\circ$

16 In the diagram, a flowerpot of mass m kg is hung from a ceiling by two chains. Let the tensions in the chains AP and BP be T_1 and T_2 newtons respectively. The third force acting at P is the weight of the flowerpot.



a By finding the horizontal component of the resultant of the three forces acting at P, show that $T_1 = \sqrt{3}\,T_2$.

b By finding the vertical component of the resultant of the three forces acting at P, show that $\sqrt{3}\,T_1 + T_2 = 19.6m$ newtons.

c Find the mass of the flowerpot, given that $T_2 = 98$ N.

a) we must have $T_1 \sin 30 = T_2 \sin 60$

$\Leftrightarrow \quad T_1 \times \frac{1}{2} = T_2 \times \frac{\sqrt{3}}{2}$ or $T_1 = \sqrt{3}\,T_2$

b) Adding the 3 forces on the vertical:

$W = T_1 \cos 30 + T_2 \cos 60$

$\Leftrightarrow \quad W = T_1 \times \frac{\sqrt{3}}{2} + T_2 \times \frac{1}{2}$

$\Leftrightarrow \quad W = \frac{1}{2}\left[\sqrt{3}\,T_1 + T_2\right]$. so $\sqrt{3}\,T_1 + T_2 = 2W$

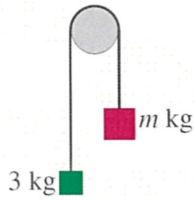
$\therefore \quad \sqrt{3}\,T_1 + T_2 = 2m \times 9.8 = 19.6\,m$

c) $m = \dfrac{\sqrt{3}\,T_1 + T_2}{19.6} = \dfrac{\sqrt{3} \times \sqrt{3}\,T_2 + T_2}{19.6} = \dfrac{4T_2}{19.6}$

$\therefore \quad m = \dfrac{4 \times 98}{19.6} = 20$ kg

APPLICATIONS TO PHYSICAL SITUATIONS (from CAMBRIDGE)

17 The diagram shows objects of mass 3 kg and m kg attached to the ends of a light inextensible string that passes over a smooth pulley. The 3 kg object is accelerating at 4.9 $\mathrm{m/s^2}$ upwards. Let the tension in the string be T newtons.



a Find the value of T.

b Find the value of m.

a) $m_3 a = T - m_3 g$ by adding the force on the vertical.

$\therefore T = m_3 a + m_3 g$

$T = m_3(a+g) = 3(4.9 + 9.8) = 44.1$ Newtons

b) The mass m must also be accelerating at $4.9\,\mathrm{ms^{-2}}$ but downwards.

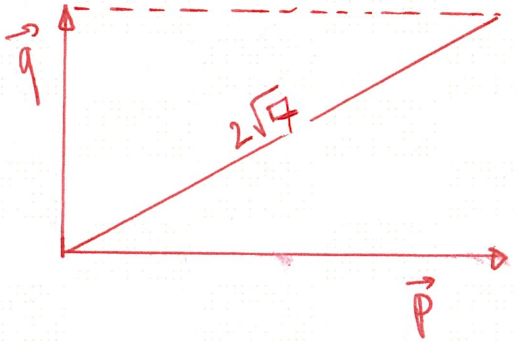
So we must have $m \times 4.9 = m \times g - T$

$\therefore m(4.9 - g) = -T$

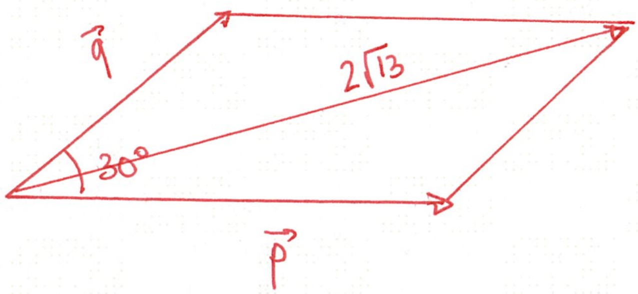
$\therefore m = \dfrac{T}{g - 4.9} = \dfrac{44.1}{9.8 - 4.9} = 9$ kg

APPLICATIONS TO PHYSICAL SITUATIONS (from CAMBRIDGE)

18 Two forces, of magnitude p newtons and q newtons, have a resultant of $2\sqrt{7}$ N when they act at 90° to each other. When they act at 30° to each other, however, the magnitude of the resultant is $2\sqrt{13}$ N. Find the values of p and q.



So $|\vec{p}|^2 + |\vec{q}|^2 = [2\sqrt{7}]^2 = 28$ ①



$$|\vec{p}+\vec{q}| = \sqrt{(|\vec{p}|+|\vec{q}|\cos 30)^2 + (|\vec{q}|\sin 30)^2}$$

Squaring both sides, we obtain:

$$(2\sqrt{13})^2 = \left[|\vec{p}| + |\vec{q}|\tfrac{\sqrt{3}}{2}\right]^2 + \left[|\vec{q}| \times \tfrac{1}{2}\right]^2 = \left[|\vec{p}|^2 + \sqrt{3}\,|\vec{p}||\vec{q}| + \tfrac{3}{4}|\vec{q}|^2\right] + \tfrac{1}{4}|\vec{q}|^2$$

$$\therefore\ 52 = |\vec{p}|^2 + |\vec{q}|^2 + \sqrt{3}\,|\vec{p}||\vec{q}|$$

$$\therefore\ 52 = 28 + \sqrt{3}\,|\vec{p}||\vec{q}|$$

$$\therefore\ |\vec{p}||\vec{q}| = \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

$$(|\vec{p}|^2 - |\vec{q}|^2)^2 = (|\vec{p}|^2 + |\vec{q}|^2)^2 - 4|\vec{p}|^2|\vec{q}|^2$$

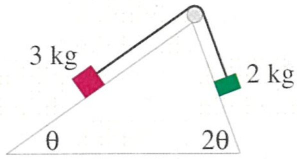
$$= 28^2 - 4(8\sqrt{3})^2 = 16$$

$\therefore\ |\vec{p}|^2 - |\vec{q}|^2 = 4$ ② So adding ① and ② gives

$2|\vec{p}|^2 = 32$ $\quad |\vec{p}|^2 = 16$ so $|\vec{p}| = 4$

and hence $|\vec{q}| = \frac{8\sqrt{3}}{4} = 2\sqrt{3}$

19 The diagram shows objects of mass 3 kg and 2 kg on connected smooth planes inclined at angles of θ and 2θ to the horizontal. The objects are attached to the ends of a light inextensible string that passes over a smooth pulley. Let T newtons be the tension in the string, and suppose that the 3 kg object is accelerating at a m/s^2 up its plane.

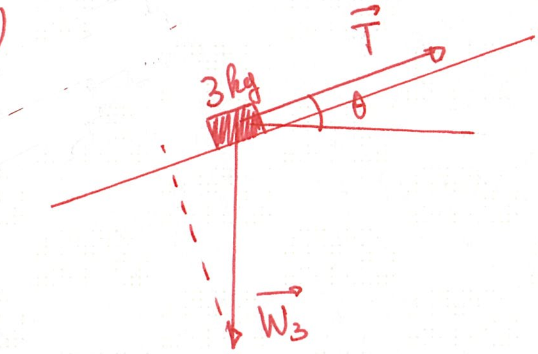


a Find, in terms of a, T, g and θ, an equation for the motion of the 3 kg object up its plane.

b Write down a similar equation for the motion of the 2 kg object down the other plane.

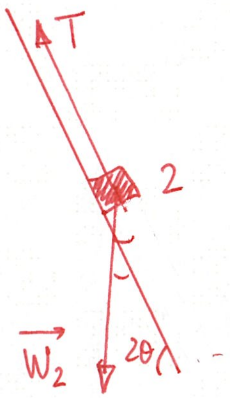
c Show that the system is in equilibrium when $\cos\theta = \frac{3}{4}$.

a)



Projecting the force on the sliding plane gives $3a = -3g\sin\theta + T_1$

b)



$$2a = -T_2 + 2g\sin 2\theta$$

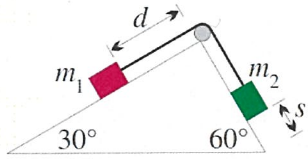
c) At equilibrium, we must have $T_1 = T_2$ and $a = 0$.

$$\therefore \begin{cases} 0 = -3g\sin\theta + T \\ 0 = -T + 2g\sin 2\theta \end{cases} \iff \begin{cases} T = 3g\sin\theta \\ T = 2g\sin 2\theta \end{cases}$$

So $3g\sin\theta = 2g\sin 2\theta \iff \frac{3}{2}\sin\theta = 2\sin\theta\cos\theta$

so $2\cos\theta = \frac{3}{2}$ hence $\cos\theta = \frac{3}{4}$

20 In the diagram, objects of mass m_1 and m_2 are held at rest on adjoining smooth inclined planes. They are connected by a light inextensible string that passes over a smooth pulley.

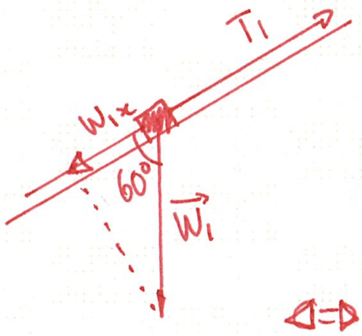


a Show that when the objects are released, the object of mass m_1 will accelerate *towards the pulley* if $m_1 < \sqrt{3}\,m_2$.

b Assuming that the condition in part **a** is satisfied, show that the acceleration of m_1 will be

$$a = \frac{g\left[\sqrt{3}\,m_2 - m_1\right]}{2(m_1 + m_2)}$$

a) The mass m_1 will accelerate towards the pulley if the projection of its weight on the sliding plane is <u>less</u> than the projection of the weight of particle 2 on its respective sliding plane.

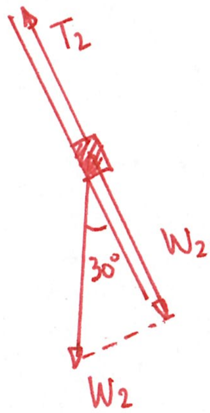


i.e. if

$$m_1 g \cos 60 < m_2 g \cos 30$$

$$\Leftrightarrow m_1 \times \frac{1}{2} < m_2 \times \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow m_1 < m_2\sqrt{3}$$



b) $m_1 a_1 = T_1 - m_1 g \cos 60$ AND $m_2 a_2 = m_2 g \cos 30 - T_2$

But $T_1 = T_2$ and $a_1 = a_2$, therefore:

$m_1 a = T - m_1 g \times \frac{1}{2}$ and $m_2 a = \frac{m_2 g \sqrt{3}}{2} - T$

Adding the two equations results in:

$$a(m_1 + m_2) = \frac{g}{2}(\sqrt{3}m_2 - m_1) \Leftrightarrow a = \frac{g(\sqrt{3}m_2 - m_1)}{2(m_1 + m_2)}$$

So $$a = \frac{g\left[\sqrt{3}\,m_2 - m_1\right]}{2(m_1 + m_2)}$$