GEOMETRICAL REPRESENTATION OF A COMPLEX NUMBER AS A VECTOR

1 On an Argand diagram, point A represents the complex number α. Point B is located so that the vector $\overrightarrow{OB}$ is the result of rotating $\overrightarrow{OA}$ anticlockwise by $\frac{2\pi}{3}$ and then halving its length. Which complex number represents point B?

A $\frac{\pi}{3}\alpha$ **B** $\alpha\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ **C** $\alpha\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ **D** $\frac{\alpha}{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

2 On a complex plane, P represents $z = -3 + 4i$ and Q represents the complex number w. Find w so that triangle OPQ is:

(a) an isosceles right-angled triangle with the right angle at O

(b) an isosceles right-angled triangle with the right angle at P

(c) right-angled at O, with OQ twice the length of OP.

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3 Point E is the centre of a square $ABCD$ (labelled anticlockwise) on an Argand diagram. E and A are the points corresponding to $-2 + i$ and $1 + 5i$ respectively. Find the complex numbers represented by the points B, C and D.

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4 **(a)** If $z_1 = 6 + 8i$ and $|z_2| = 15$, show that the greatest possible value of $|z_1 + z_2|$ is 25.

(b) If $|z_1 + z_2|$ takes this greatest value, find z_2 in Cartesian form.

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5 On an Argand diagram, P represents $z = 1 + i$ and Q represents q. Find the two possible values of q (in mod–arg form) such that ΔOPQ is equilateral.

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8 z_1 and z_2 are two complex numbers of equal moduli, with $\arg z_1 = \theta_1$ and $\arg z_2 = \theta_2$. Use an Argand diagram to find the values of $\arg(z_1 + z_2)$ and $\arg(z_1 - z_2)$ in terms of θ_1 and θ_2.

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9 The points P and Q in the complex plane correspond to the complex numbers z and w respectively. Triangle OPQ is right-angled and isosceles with $OP = OQ$.

(a) Show that $w^2 + z^2 = 0$.

(b) If $OPRQ$ is a square, find (in terms of z) the complex number represented by E, the point of intersection of the diagonals of the square.

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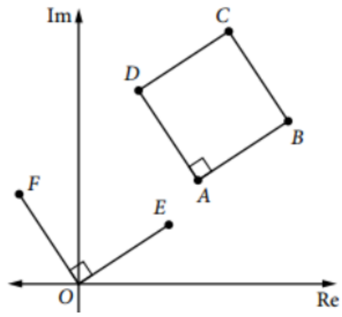
10 On an Argand diagram, $ABCD$ is a square. OE and OF are parallel to and equal in length to AB and AD respectively. The vertices A and B correspond to the complex numbers w_1 and w_2 respectively.

(a) Explain why the point E corresponds to $w_2 - w_1$.

(b) What complex number corresponds to the point F?

(c) What complex number corresponds to the vertex D?



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11 z_1 and z_2 are two complex numbers such that $\frac{z_1 + z_2}{z_1 - z_2} = 2i$.

(a) Show that $|z_1| = |z_2|$.

(b) If α is the angle between the vectors representing z_1 and z_2, show that $\tan\frac{\alpha}{2} = \frac{1}{2}$

(c) Show that $z_2 = \frac{1}{5}(3 + 4i)z_1$.