PIGEONHOLE PRINCIPLE

The pigeonhole principle is a way of understanding how a number of items can be placed into a number of containers.

Traditionally, mathematicians have illustrated this concept by using the example of placing pigeons into pigeonholes. For example, if you have five pigeons, and four pigeonholes, then one of the pigeonholes must contain at least two pigeons.

If you place one pigeon in each pigeonhole, then the only way that you can place the remaining pigeon is to put it in with one of the pigeons already in a pigeonhole.

Pigeonhole principle: if $(n + 1)$ items occupy n containers, then at least one of the containers must contain at least two items.

Alternative statement: if n items are sitting in k containers, where $n > k$, then there is at least one container with at least n/k items in it.

Examples

a) You have a drawer full of red and blue socks that have not been arranged in pairs. Socks can be worn on either foot. What is the least number of socks that you must take from the drawer to be sure that you have one pair of the same colour?

In this example the socks are the items and the colour of the socks are the containers. Thus there are two containers, so you need three items to be sure that you have two items in the same container.

Hence you must select three socks from the drawer to be sure that you have one pair of the same colour. You would have either three socks of the same colour or two socks of the same colour and the other sock the remaining colour.

b) Explain why any 27-word sequence in this book must have at least two words that start with the same letter.

There are 27 words (items) that can start with one of the 26 different letters of the alphabet (containers). By the pigeonhole principle, two of the words must start with the same letter.

Example 6

There are seven pigeons sitting in three pigeonholes.

Explain why one of the pigeonholes must contain at least three pigeons.

Solution

$n = 7, k = 3$, so $\frac{n}{k} = \frac{7}{3} = 2\frac{1}{3}$

Hence one pigeonhole must contain at least $2\frac{1}{3}$ pigeons, but since part-pigeons are impossible, it must contain three pigeons.