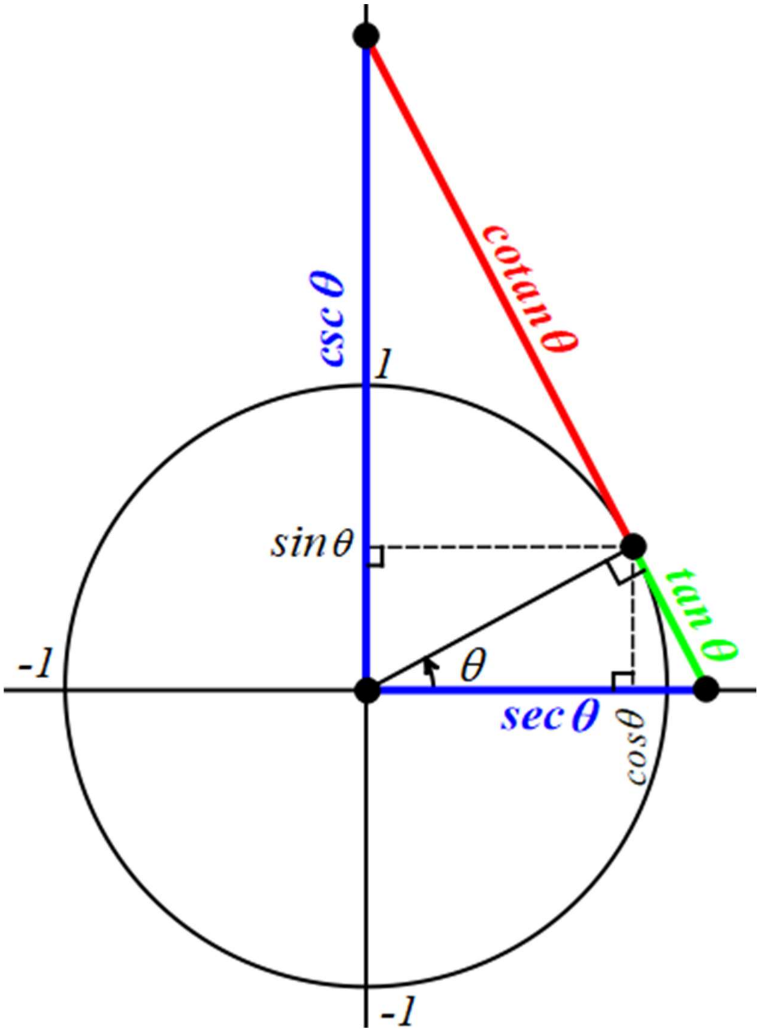
INVERSE TRIGONOMETRIC RATIOS



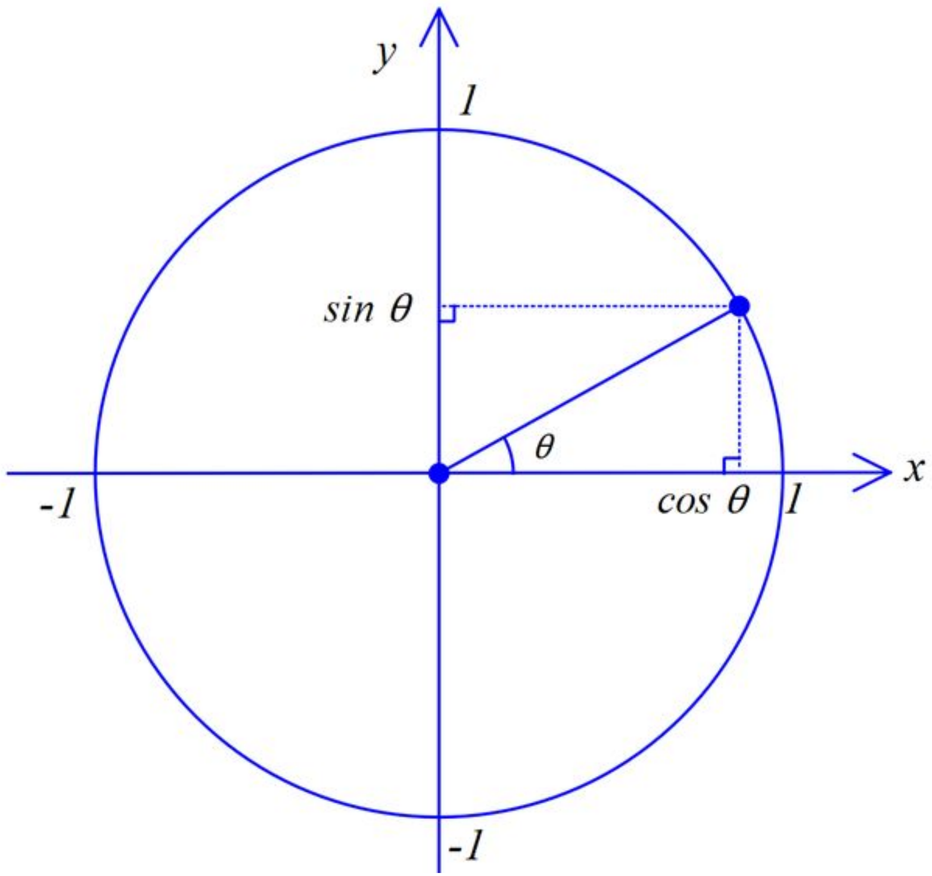
Using similar triangles, it is relatively easy to demonstrate that:

tangent (abbrev. "*tan*")	$tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$
cotangent (abbrev. "*cotan*" or "*cot*")	$cot\,\theta = \frac{cos\,\theta}{sin\,\theta}$
secant (abbrev. "*sec*")	$sec\,\theta = \frac{1}{cos\,\theta}$
cosecant (abbrev. "*cosec*" or "*csc*")	$csc\,\theta = \frac{1}{sin\,\theta}$

Further, using Pythagoras, we obtain, in addition to $sin^2\,\theta + cos^2\,\theta = 1$

$$\mathbf{1 + tan^2\,\theta = sec^2\,\theta} \qquad \mathbf{1 + cot^2\,\theta = csc^2\,\theta}$$

THE UNIT CIRCLE - sine and cosine



- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{y}{1} = y$
- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{x}{1} = x$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x} = \dfrac{\sin\theta}{\cos\theta}$

Instead of ratios, in the unit circle, cos and sin are respectively the **x and y coordinates** of the intersection of the radius with the circle.

QUADRANTS

2nd quadrant

$\sin\theta > 0$

$\cos\theta < 0$

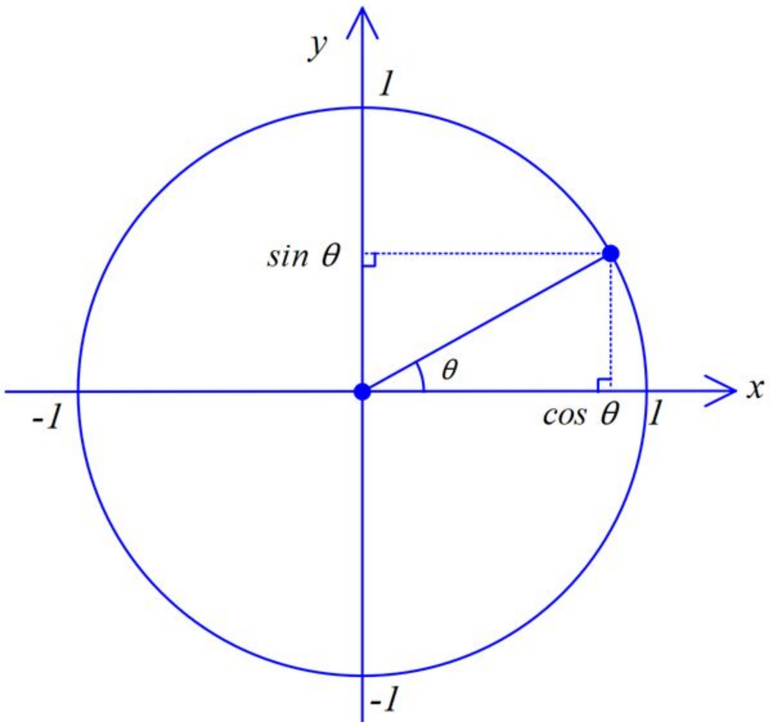
$\tan\theta < 0$

1st quadrant

$\sin\theta > 0$

$\cos\theta > 0$

$\tan\theta > 0$



3rd quadrant

$\sin\theta < 0$

$\cos\theta < 0$

$\tan\theta > 0$

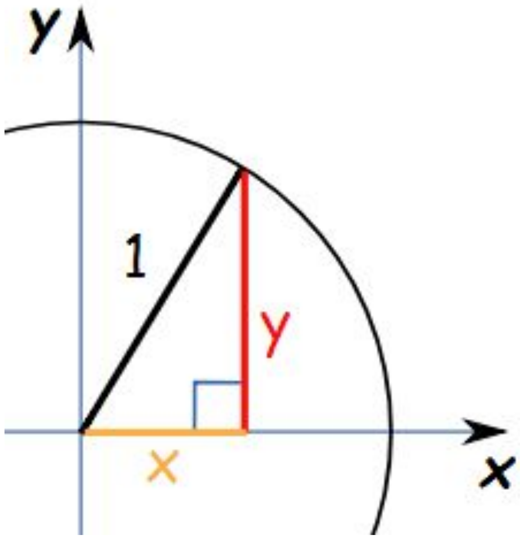
4th quadrant

$\sin\theta < 0$

$\cos\theta > 0$

$\tan\theta < 0$

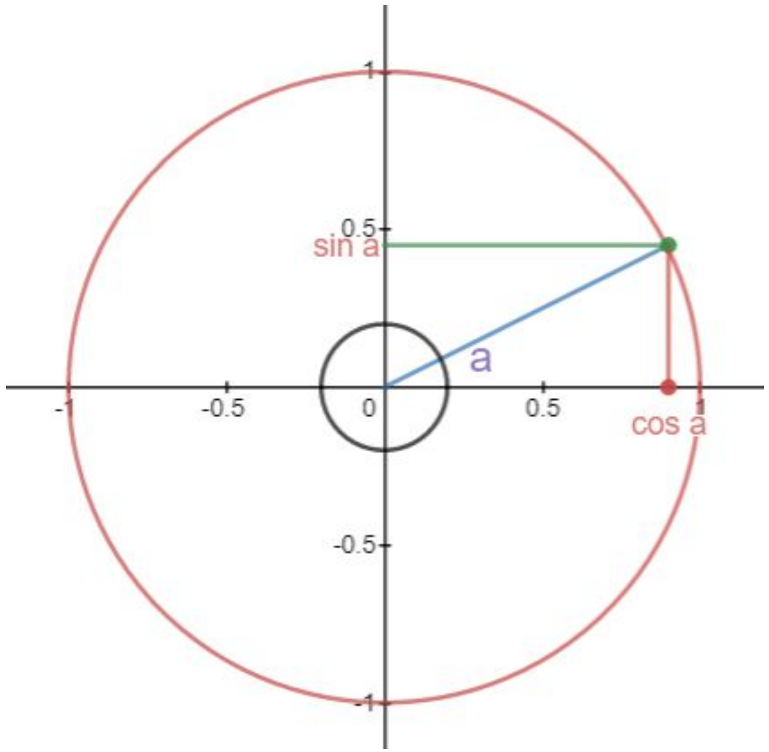
RELATIONSHIPS BETWEEN THE RATIOS



By applying Pythagoras theorem to this right-angle triangle, we get:

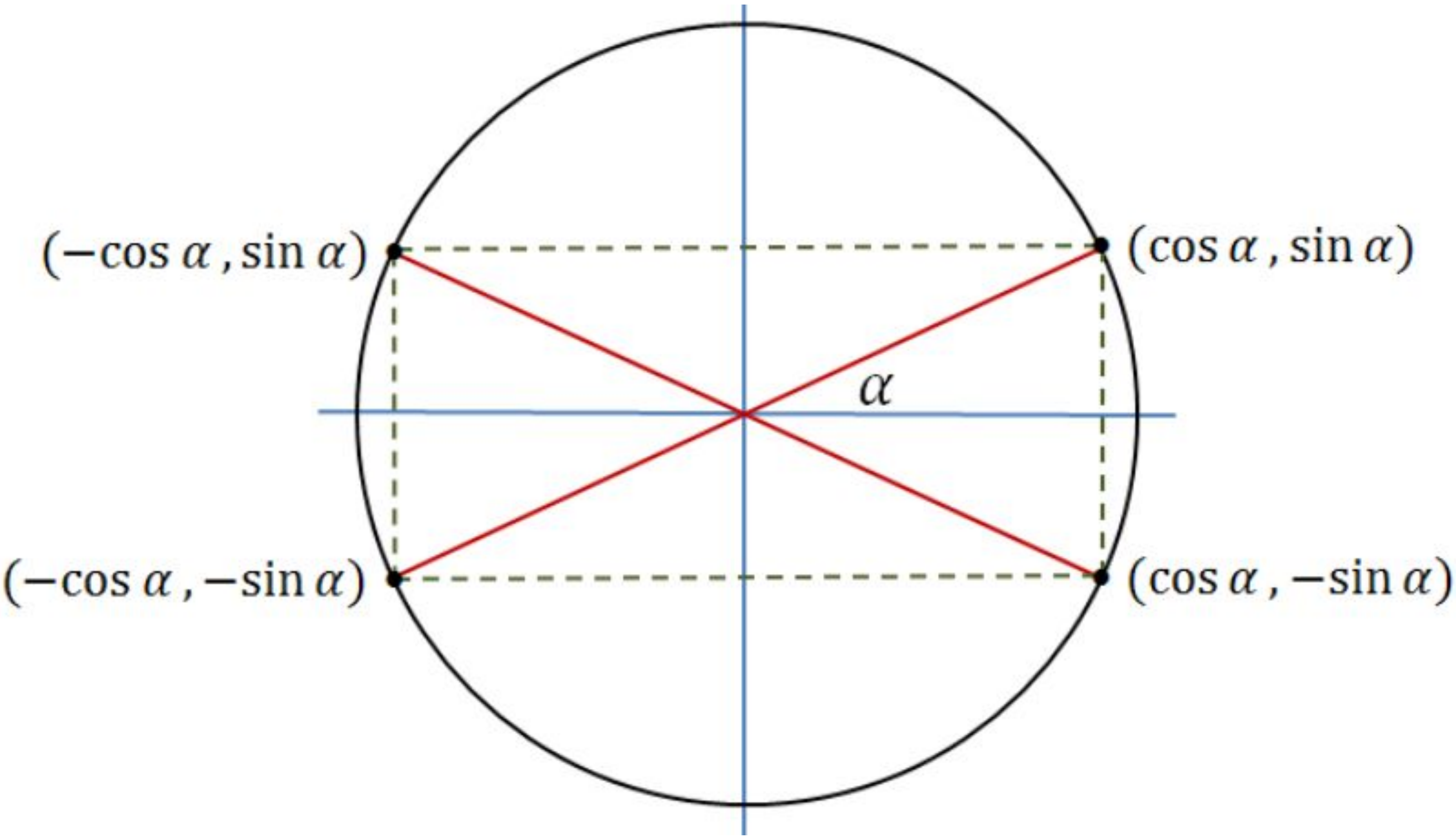
$$cos^2\theta + sin^2\theta = 1$$

SIN and COS are PERIODIC (PERIOD is 360°)



$$\sin(\theta + 360) = \sin\theta \qquad \cos(\theta + 360) = \cos\theta$$

RELATIONSHIPS BETWEEN TRIGONOMETRIC RATIOS FOR (-θ), (180 + θ), (180 - θ)



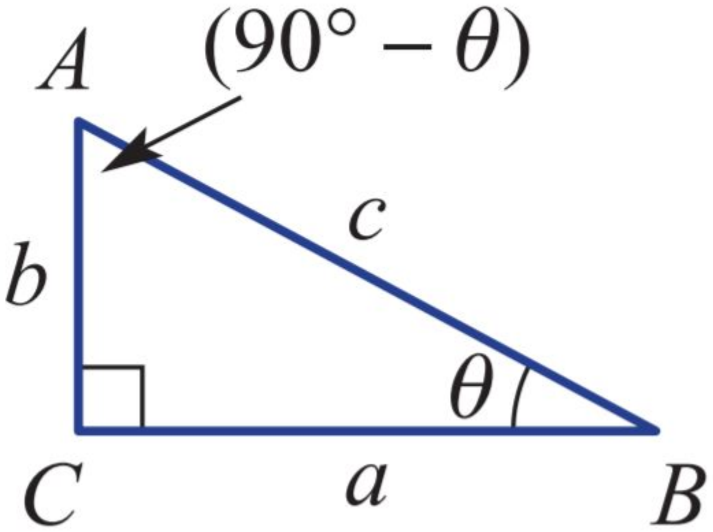
TRIGONOMETRIC RATIOS - $(-\theta)$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

RELATIONSHIPS BETWEEN TRIGONOMETRIC RATIOS FOR COMPLEMENTARY ANGLES



$$\sin(90-\theta)=\frac{a}{c}=\cos\theta$$

$$\cos(90-\theta)=\frac{b}{c}=\sin\theta$$

$$tan(90-\theta)=\frac{sin(90-\theta)}{cos(90-\theta)}=\frac{cos\,\theta}{sin\,\theta}=\frac{1}{tan\,\theta}$$

TRIGONOMETRIC RATIOS - (180 - θ) and (180 + θ)

$$\sin(180-\theta) = \sin\theta$$

$$\cos(180-\theta) = -\cos\theta$$

$$\tan(180-\theta) = \frac{\sin(180-\theta)}{\cos(180-\theta)} = \frac{\sin\theta}{-\cos\theta} = -\tan\theta$$

$$\sin(180+\theta) = -\sin\theta$$

$$\cos(180+\theta) = -\cos\theta$$

$$\tan(180+\theta) = \frac{\sin(180+\theta)}{\cos(180+\theta)} = \frac{-\sin\theta}{-\cos\theta} = \tan\theta$$