QUADRATIC EQUATIONS WITH NON-RATIONAL SOLUTIONS

Complete the square to solve the following quadratic equations, giving answers to even-numbered questions in surd form and answers to odd-numbered questions correct to two decimal places.

1 $x^2 - 2x - 4 = 0$ 2 $x^2 + 4x - 4 = 0$ 3 $x^2 - x - 5 = 0$ 4 $x^2 - 6x + 2 = 0$

① $\Leftrightarrow (x-1)^2 - 1 = 4 \Leftrightarrow x - 1 = \pm\sqrt{5} \Leftrightarrow x = \pm\sqrt{5} + 1$

so $x = \sqrt{5}+1$ or $x = +1 - \sqrt{5}$

② $\Leftrightarrow (x+2)^2 - 4 - 4 = 0 \Leftrightarrow (x+2)^2 = 8$

so $x + 2 = \pm\sqrt{8} = \pm 2\sqrt{2}$

so $x = \pm 2\sqrt{2} - 2$

either $x = 2\sqrt{2} - 2$ or $x = -2\sqrt{2} - 2$

③ $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 5 = 0 \Leftrightarrow \left(x - \frac{1}{2}\right)^2 = \frac{21}{4}$

so $x - \frac{1}{2} = \pm\sqrt{\frac{21}{4}} = \pm\frac{\sqrt{21}}{2} \Leftrightarrow x = \pm\frac{\sqrt{21}}{2} + \frac{1}{2}$

either $x = \frac{1+\sqrt{21}}{2}$ or $x = \frac{1-\sqrt{21}}{2}$

④ $(x-3)^2 - 9 + 2 = 0 \Leftrightarrow (x-3)^2 = 7$

$x - 3 = \pm\sqrt{7} \Leftrightarrow x = \pm\sqrt{7} + 3$

either $x = 3 + \sqrt{7}$ or $x = 3 - \sqrt{7}$

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9 $x^2-6x-5=0$ 10 $x^2+4x=1$ 11 $x^2=2x+5$ 12 $x^2+3x-6=0$

(9) $(x-3)^2-9-5=0 \iff (x-3)^2=14$

so $x-3=\pm\sqrt{14} \iff x=\pm\sqrt{14}+3$

either $x=3+\sqrt{14}$ or $x=3-\sqrt{14}$

(10) $(x+2)^2-4=1 \iff (x+2)^2=5$

so $x+2=\pm\sqrt{5} \iff x=\pm\sqrt{5}-2$

either $x=\sqrt{5}-2$ or $x=-2-\sqrt{5}$

(11) $\iff x^2-2x=5 \iff (x-1)^2-1=5$

$\iff (x-1)^2=6 \iff x-1=\pm\sqrt{6}$

so either $x=\sqrt{6}+1$ or $x=1-\sqrt{6}$

(12) $\iff \left(x+\frac{3}{2}\right)^2-\frac{9}{4}-6=0$

$\iff \left(x+\frac{3}{2}\right)^2=\frac{33}{4}$

$\left(x+\frac{3}{2}\right)=\pm\sqrt{\frac{33}{4}}=\pm\frac{\sqrt{33}}{2}$

so either $x=\frac{\sqrt{33}}{2}-\frac{3}{2}$ or $x=-\frac{\sqrt{33}}{2}-\frac{3}{2}$