STANDARD FACTORISATIONS

Factorising using the difference of two squares

$$a^2 - b^2 = (a - b)(a + b)$$

Example 5

Factorise:

(a) $a^2 - 25$ (b) $9x^2 - 49$ (c) $(x+1)^2 - (y-1)^2$ (d) $a^3 - a^2 b - ab^2 + b^3$

Solution

(a) $a^2 - 25 = a^2 - 5^2$
$\qquad = (a - 5)(a + 5)$

(b) $9x^2 - 49 = (3x)^2 - 7^2$
$\qquad = (3x - 7)(3x + 7)$

(c) $(x+1)^2 - (y-1)^2$
$\quad = [(x+1) - (y-1)][(x+1) + (y-1)]$
$\quad = (x - y + 2)(x + y)$

(d) $a^3 - a^2 b - ab^2 + b^3$
$\quad = a^2(a - b) - b^2(a - b)$
$\quad = (a^2 - b^2)(a - b)$
$\quad = (a - b)(a + b)(a - b)$
$\quad = (a - b)^2(a + b)$

Sum and difference of two cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 6

Factorise:

(a) $x^3 - 8$ (b) $27y^3 + 64x^3$ (c) $(x+2)^3 + y^3$ (d) $x^2 y^3 - z^2 y^3 - x^2 w^3 + z^2 w^3$

Solution

(a) $x^3 - 8 = x^3 - 2^3$
$\qquad = (x - 2)(x^2 + 2x + 4)$

(b) $27y^3 + 64x^3 = (3y)^3 + (4x)^3$
$\qquad = (3y + 4x)(9y^2 - 12xy + 16x^2)$

(c) $(x+2)^3 + y^3$
$\quad = (x + 2 + y)[(x+2)^2 - (x+2)y + y^2]$
$\quad = (x + 2 + y)(x^2 + 4x + 4 - xy - 2y + y^2)$

(d) $x^2 y^3 - z^2 y^3 - x^2 w^3 + z^2 w^3$
$\quad = y^3(x^2 - z^2) - w^3(x^2 - z^2)$
$\quad = (x^2 - z^2)(y^3 - w^3)$
$\quad = (x - z)(x + z)(y - w)(y^2 + yw + w^2)$