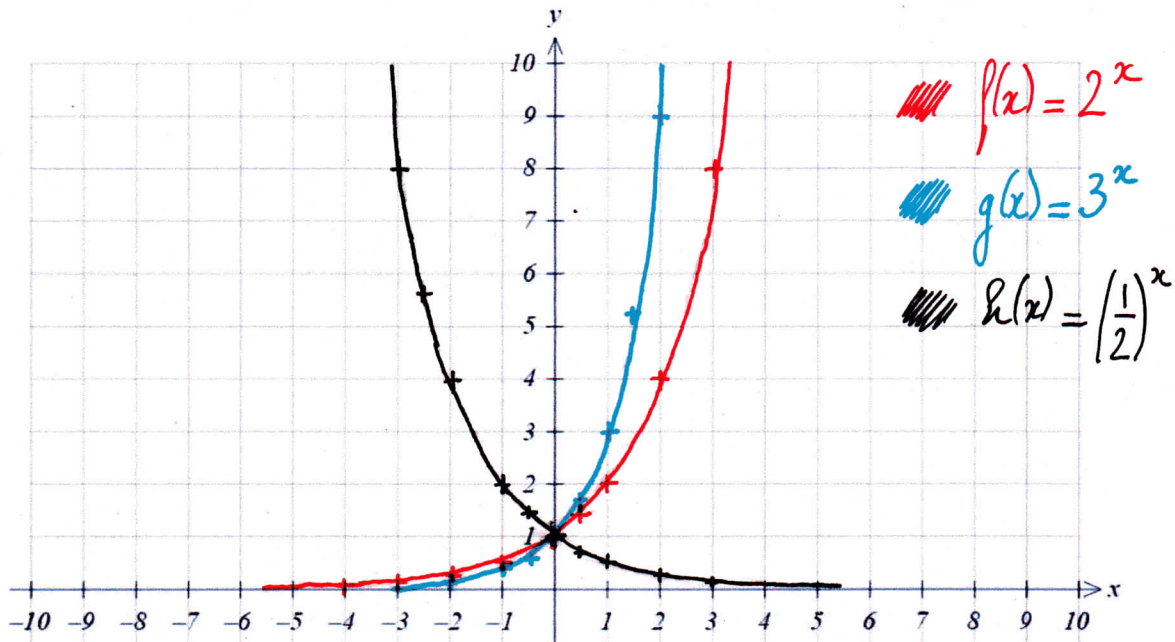# GRAPHS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

An exponential function is of the form $f(x) = a^x$ where $a > 0$ and $a \neq 1$

Example 1: $f(x) = 2^x$ We can fill a table of values:

x	-5	-4	-3	-2	-1	-0.5	0	0.5	1	2	3
y	0.03	0.06	0.125	0.25	0.5	0.71	1	1.4..	2	4	8

$f(x) = 2^x$

$g(x) = 3^x$

$h(x) = \left(\frac{1}{2}\right)^x$

Example 2: $f(x) = 3^x$ Fill a table of values, then draw the function above

x	-5	-4	-3	-2	-1	-0.5	0	0.5	1	2	3
y	0.004	0.01	0.04	0.11	0.33	0.58	1	1.7	3	9	27

Example 3: $f(x) = \left(\frac{1}{2}\right)^x$ Fill a table of values, then draw the function above

| x	-5	-4	-3	-2	-1	-0.5	0	0.5	1	2	3
y	32	16	8	4	2	1.4..	1	0.71	0.5	0.25	0.125

GRAPHS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Observations:

- All exponential functions:
  - pass through the point (0,1)
  - have $y = 0$ as an asymptote.

- For all exponential functions, the **domain** is $\mathbb{R}$ (all real numbers) and the **range** is $\mathbb{R}^+$.

- The larger the value of "a", the steeper the curve.

- The value of "a" for which the gradient of the curve at Point (0,1) is exactly 1 is 2.718281828.... This number is named e (from the mathematician Leonhard Euler who studied it extensively). This very special number continues indefinitely and never repeats, so it is an "*irrational*" number. Like the number π, it is also said to be "*transcendental*" as it cannot be a solution to a polynomial equation with rational coefficients.

The inverse function of $f(x) = e^x$ is $f^{-1}(x) = \ln(x)$ so $e^{\ln(x)} = \ln(e^x) = x$

Because these the functions $f(x) = \ln(x)$ and $g(x) = e^x$ are inverse of each other, their graphs are symmetrical about the line $y = x$, as shown on the graph below:
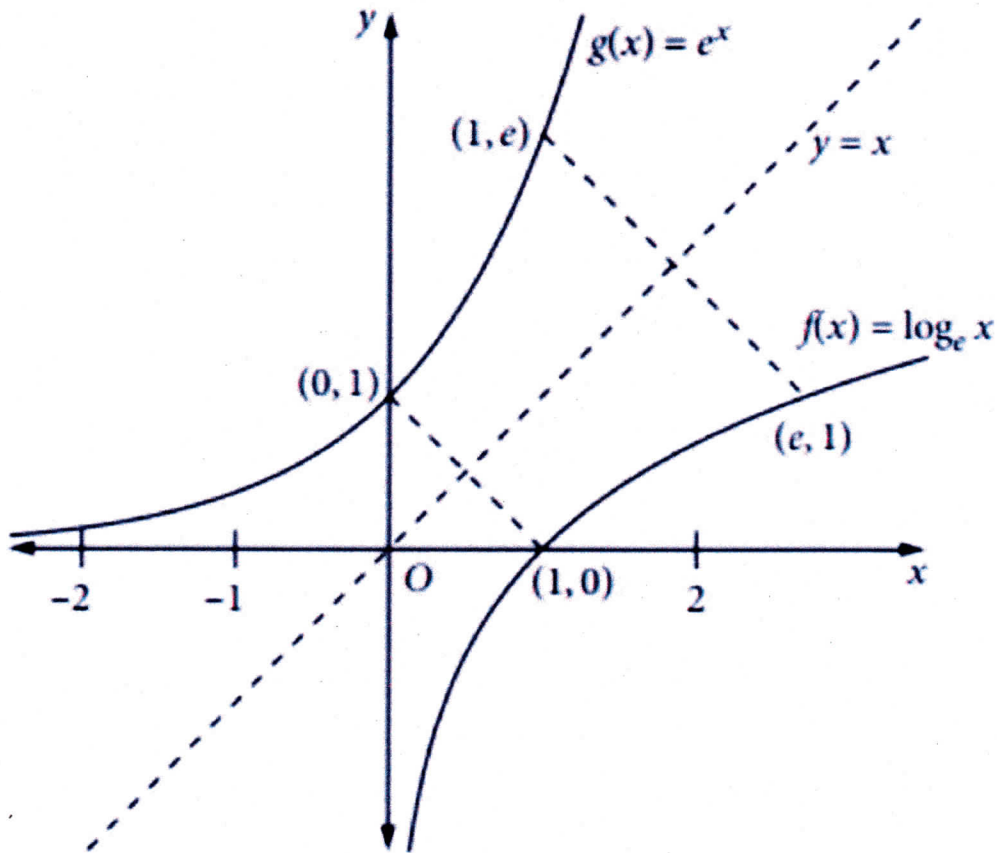
GRAPHS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

On the graph below, graph the functions $f(x) = e^x$ and $g(x) = \ln(x)$.

Note that because the gradient of the graph of $f(x) = e^x$ at the point (0,1) is 1 (by definition of the number e), the gradient of the graph of $f(x) = \ln(x)$ at the point (1,0), i.e. symmetrical of the point (0,1) with regard to the line $y = x$, is also 1. These gradients have been marked in red on the blank graph below.
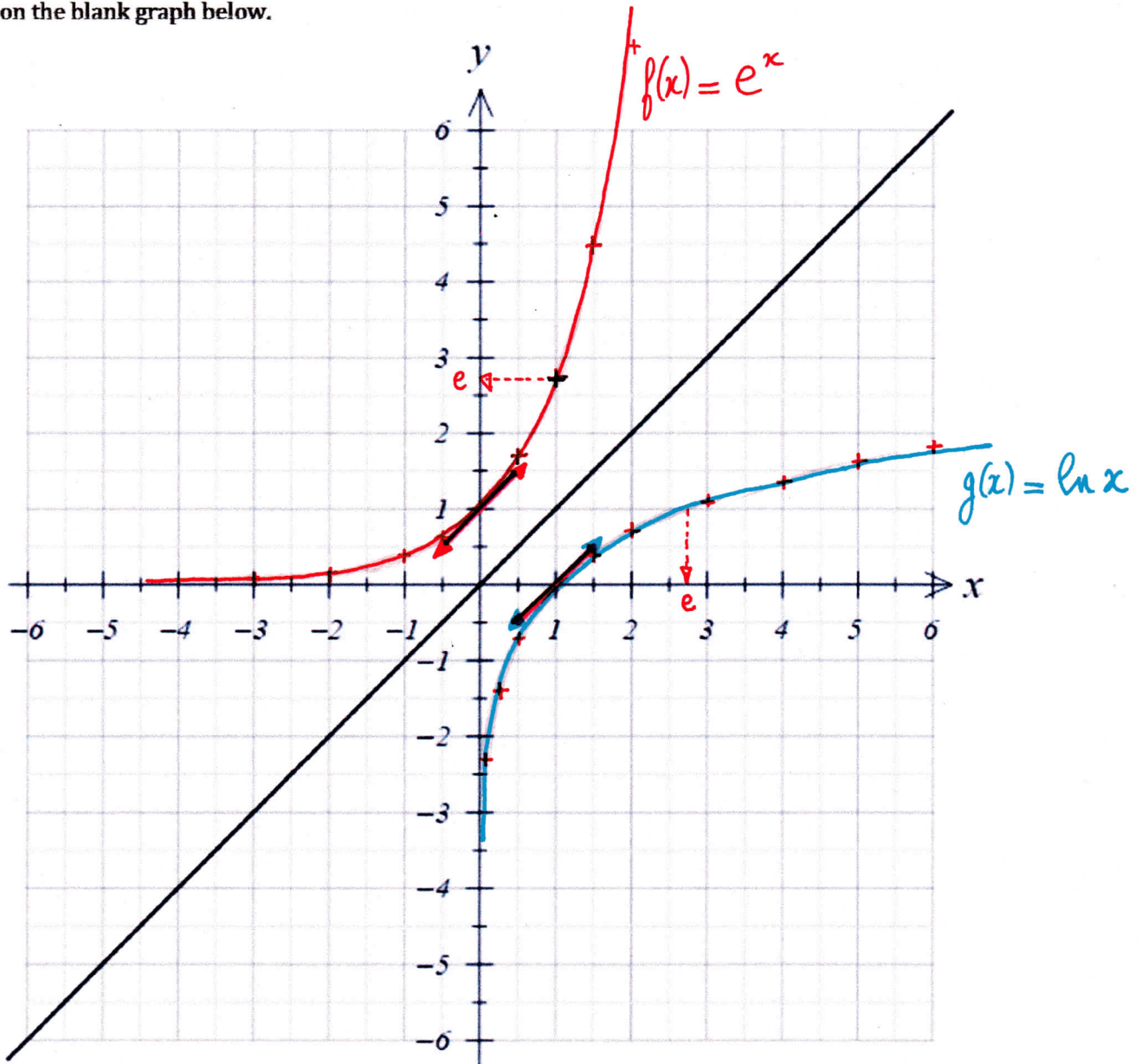
GRAPHS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

On the graph below, graph the function $f(x) = e^{-x}$

The tangent at point (0,1) has been marked in red to assist graphing.

gradient at $(0,1)$ is -1