

ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

When adding and subtracting fractions, first rewrite each fraction with the same common denominator, then add or subtract the numerators.

Example 13

Express as a single fraction:

(a) $\frac{3}{5} - \frac{2}{7}$

(b) $\frac{3x}{5} - \frac{2x+1}{7}$

(c) $\frac{3}{5x} + \frac{2}{7x}$

(d) $\frac{2x-y}{3} - \frac{x-y}{6}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{3}{5} - \frac{2}{7} &= \frac{21}{35} - \frac{10}{35} \\ &= \frac{11}{35} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{3x}{5} - \frac{2x+1}{7} &= \frac{21x}{35} - \frac{5(2x+1)}{35} \\ &= \frac{21x-10x-5}{35} \\ &= \frac{11x-5}{35} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \frac{3}{5x} + \frac{2}{7x} &= \frac{21}{35x} + \frac{10}{35x} \\ &= \frac{31}{35x} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \frac{2x-y}{3} - \frac{x-y}{6} &= \frac{2(2x-y)}{6} - \frac{(x-y)}{6} \\ &= \frac{4x-2y-x+y}{6} \\ &= \frac{3x-y}{6} \end{aligned}$$

Harder algebraic fractions

More complex algebraic fractions require you to factorise the denominator before you find the common denominator.

Write each fraction with the common denominator before you add or subtract the numerators.

Example 14

Express as a single fraction:

(a) $\frac{3}{x^2-4} + \frac{1}{x-2}$

(b) $\frac{1}{x-y} - \frac{1}{x+y}$

(c) $\frac{3}{x^2+2x} - \frac{2}{x^2-4}$

(d) $\frac{1}{x^2-5x+6} - \frac{1}{x^2+2x-8}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{3}{x^2-4} + \frac{1}{x-2} &= \frac{3}{(x-2)(x+2)} + \frac{1}{x-2} \\ &= \frac{3}{(x-2)(x+2)} + \frac{x+2}{(x-2)(x+2)} \\ &= \frac{x+5}{(x-2)(x+2)} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{1}{x-y} - \frac{1}{x+y} &= \frac{x+y}{(x-y)(x+y)} - \frac{x-y}{(x-y)(x+y)} \\ &= \frac{x+y-(x-y)}{(x-y)(x+y)} \\ &= \frac{2y}{(x-y)(x+y)} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \frac{3}{x^2+2x} - \frac{2}{x^2-4} &= \frac{3}{x(x+2)} - \frac{2}{(x-2)(x+2)} \\ &= \frac{3(x-2)}{x(x+2)(x-2)} - \frac{2x}{x(x-2)(x+2)} \\ &= \frac{3x-6-2x}{x(x+2)(x-2)} \\ &= \frac{x-6}{x(x+2)(x-2)} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \frac{1}{x^2-5x+6} - \frac{1}{x^2+2x-8} &= \frac{1}{(x-2)(x-3)} - \frac{1}{(x-2)(x+4)} \\ &= \frac{x+4}{(x-2)(x-3)(x+4)} - \frac{x-3}{(x-2)(x-3)(x+4)} \\ &= \frac{x+4-(x-3)}{(x-2)(x-3)(x+4)} \\ &= \frac{x+4-x+3}{(x-2)(x-3)(x+4)} \\ &= \frac{7}{(x-2)(x-3)(x+4)} \end{aligned}$$