INTEGRALS INVOLVING TRIGONOMETRIC SUBSTITUTIONS

1 Evaluate $\int_{-4}^{4} \sqrt{16-x^2}\,dx$ using the substitution $x = 4\sin\theta$. What shape have you just found the area of?

2 Evaluate $\int_{0}^{1} x\sqrt{1-x^2}\,dx$ using the substitution $x = \sin\theta$.

3 Evaluate $\int_0^2 \frac{dx}{4+x^2}$ using the substitution $x = 2\tan\theta$.

6 Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx$ using the substitution $x = \cos\theta$.

9 Evaluate $\int_0^4 x\sqrt{16-x^2}\,dx$ using the substitution $x = 4\sin\theta$.

10 Evaluate $\int_0^{\sqrt{3}} \frac{1}{9+x^2}dx$ using the substitution $x = 3\tan\theta$.

12 Use the substitution $u = \tan x$ to find $\int \frac{e^{\tan x}}{\cos^2 x} dx$.

13 Use the substitution $u = \cos x + \sin x$ to find $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$.

14 **(a)** By writing $\sec x = \frac{1}{\cos x}$, show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(b) Using the substitution $u = 1 + \sec x$, find $\int \frac{\sec x \tan x}{1 + \sec x} dx$.