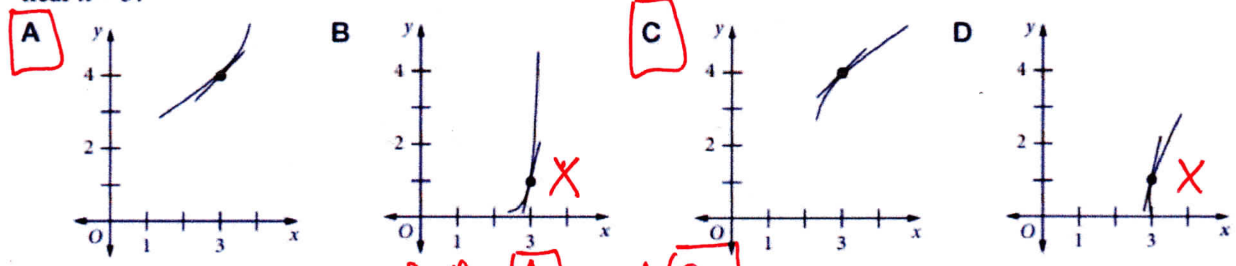
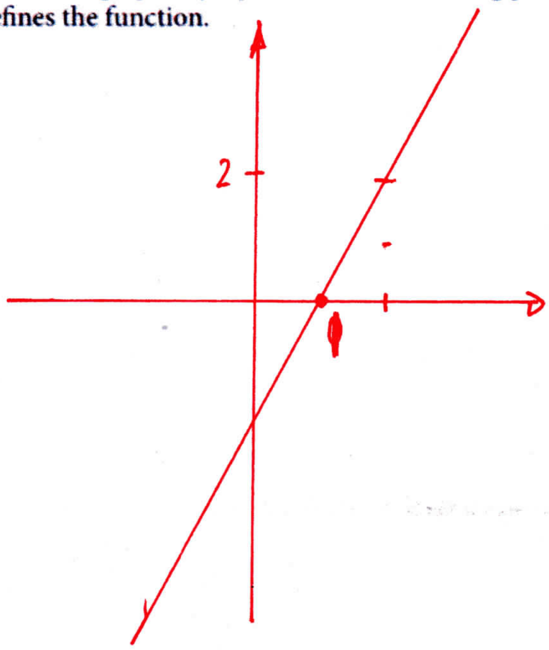
THE SIGN OF THE FIRST DERIVATIVE

1 A function $f(x)$ has the following properties: $f(3) = 4$, $f'(3) = 1$. Which sketches fit the graph of $y = f(x)$ near $x = 3$?



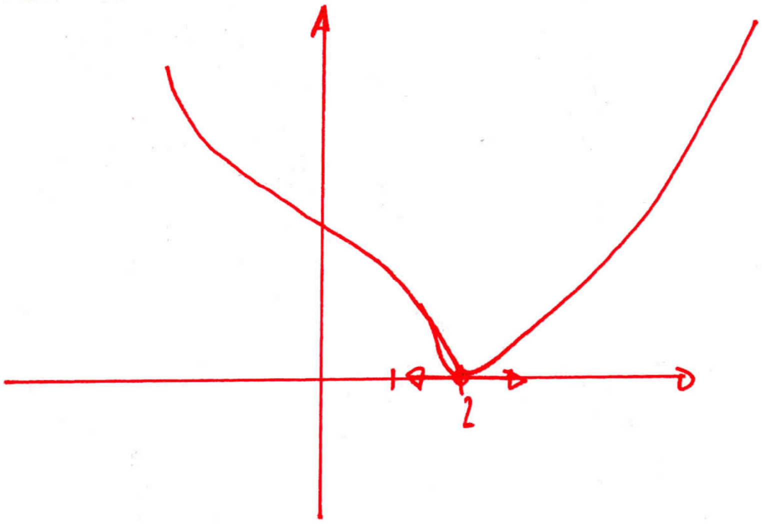
Both $\boxed{A}$ and $\boxed{C}$

2 Sketch the graph of $y = f(x)$ with the following properties: $f(1) = 0$, $f'(x) = 2$ for all x. State the rule that defines the function.



$y = 2x - 2$ line

3 Sketch the graph of a function given that $f(2) = 0$, $f'(2) = 0$, $f'(x) < 0$ for all $x < 2$, and $f'(x) > 0$ for all $x > 2$.



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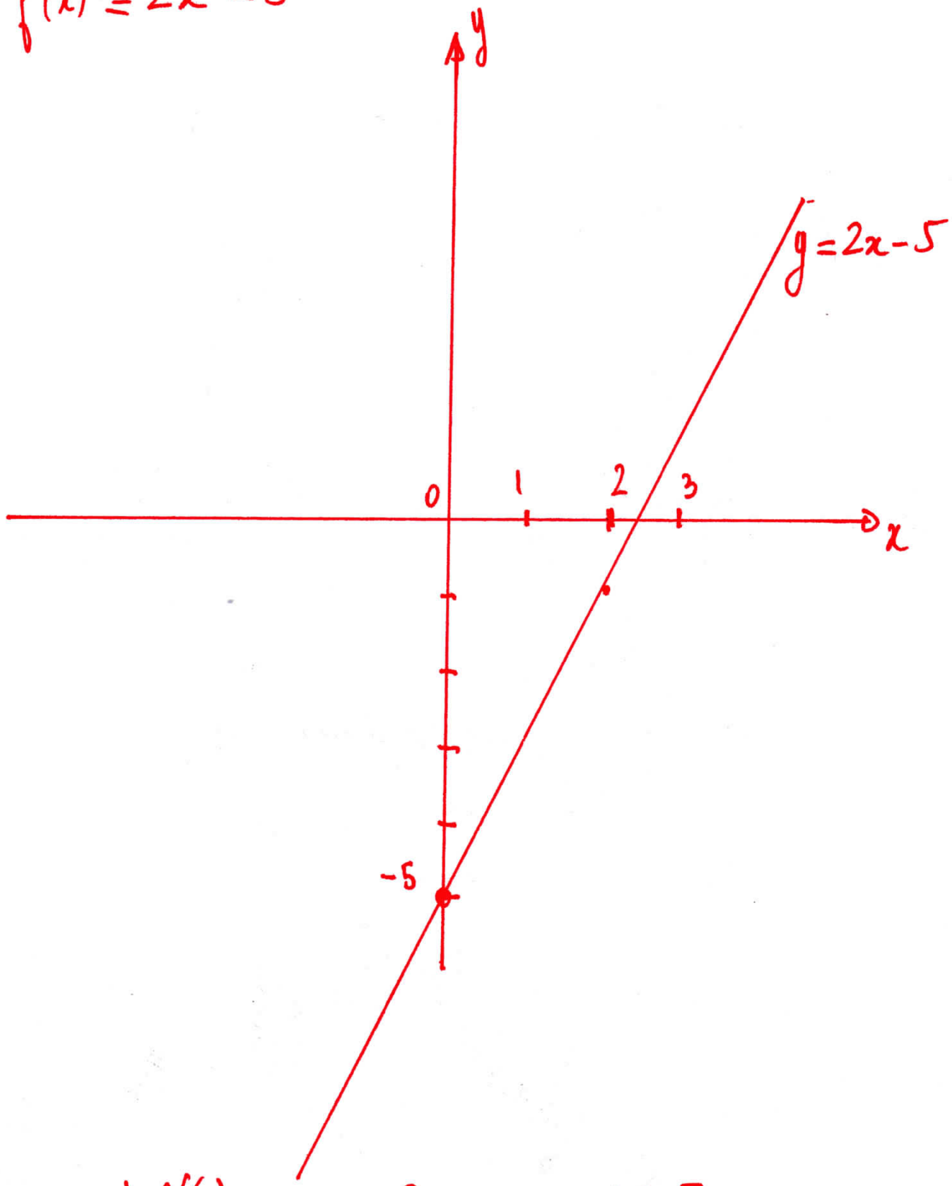
5 For the function $f(x) = x^2 - 5x + 6$, sketch the graph of $f'(x)$ and hence find the values of x for which:

(a) $f'(x) < 0$ **(b)** $f'(x) = 0$ **(c)** $f'(x) > 0$.

$f'(x) = 2x - 5$



a) $f'(x) < 0$ when $x < 2.5$

b) $f'(x) = 0$ for $x = 2.5$

c) $f'(x) > 0$ for $x > 2.5$

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7 For the graph of $f(x) = 6 - 3x - x^2$, find the values of x for which the function:

(a) increases when x increases **(b)** decreases when x increases

(c) changes from increasing to decreasing.

$f'(x) = -3 - 2x$

a) $f'(x) > 0$ when $-3 - 2x > 0$

or $-3 > 2x$

i.e: when $x < -3/2$

So when $x < -3/2$, $f'(x) > 0$ and so f increases.

b) $f'(x) < 0$ when $x > -3/2$

So when $x > -3/2$, $f'(x) < 0$ and so f decreases.

c) $f'(x) = 0$ when $x = -3/2$

f then changes from increasing to decreasing.

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8 For the graph of $f(x) = x^3 - 6x^2 + 9x + 2$, find:

(a) $f'(x)$

(b) the values of x for which the function increases when x increases

(c) the values of x for which the function decreases when x increases

(d) the values of x for which the function changes from increasing to decreasing.

a) $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

b) $f'(x) > 0$ when $x^2 - 4x + 3 > 0$

$\Delta = 16 - 4 \times 3 = 4 > 0$ so two roots.

~~For $x = 0$ $f'(0) = 3 > 0$ so $f'(x)$ is always positive.~~

$x_1 = \dfrac{4+2}{2} = 3$ and $x_2 = \dfrac{4-2}{2} = 1$

So $f'(x) = 3(x-3)(x-1)$

Between 1 and 3, say $x = 2$ $f'(2) < 0$.

So $f'(x) < 0$ between 1 and 3 (so f decreasing over that interval)

and $f'(x) > 0$ outside this interval so f increasing then.

d) $f'(x) = 0$ for $x = 1$ and $x = 3$.

10 For the graph of $f(x) = (x-1)^2(x+1)$, find the values of x for which the function is:

(a) stationary **(b)** increasing **(c)** decreasing.

$f'(x) = 2(x-1)(x+1) + (x-1)^2 = (x-1)[2x+2+x-1] = (x-1)[3x+1]$

a) $f'(x) = 0$ when $x = 1$ or $x = -1/3$

b) $f'(x) > 0$ when $x > 1$ or $x < -1/3$

c) $f'(x) < 0$ when $-\frac{1}{3} < x < 1$