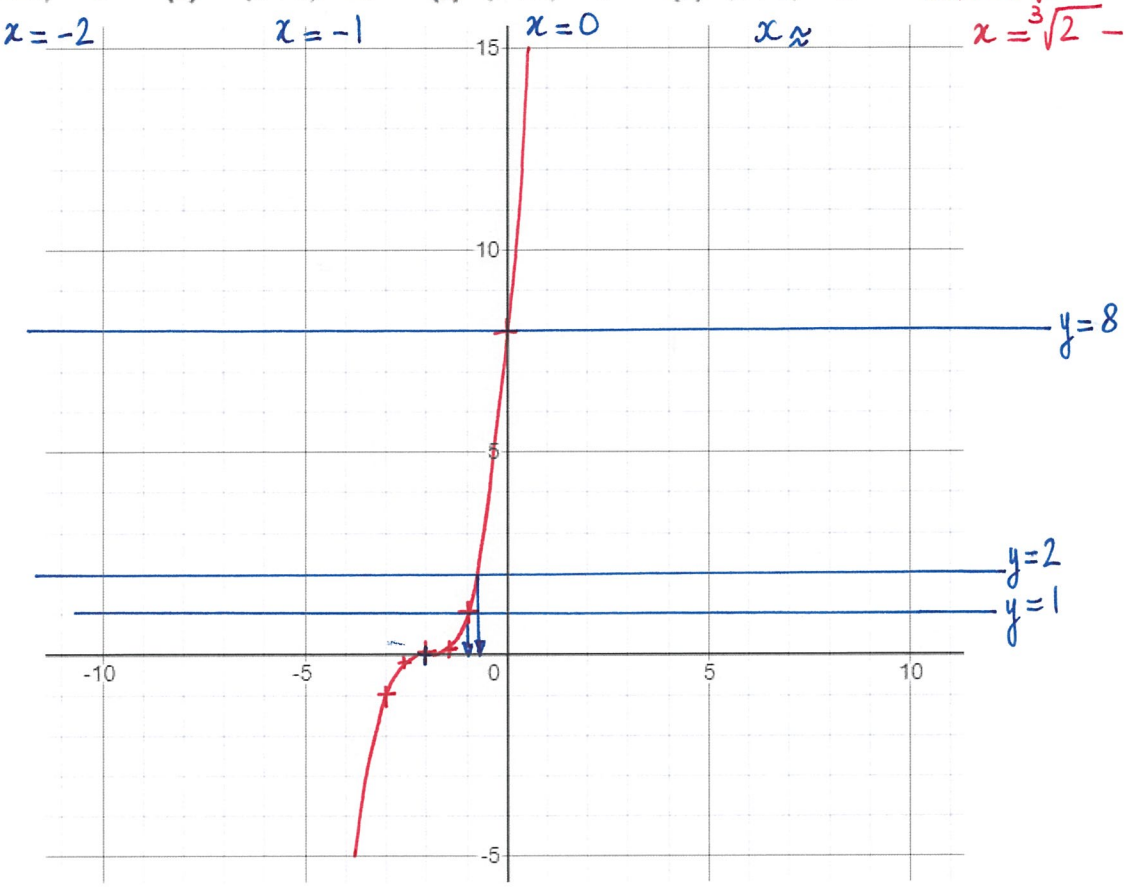
CUBIC POLYNOMIALS

1 Draw the graph of $y = (x+2)^3$. On your graph, draw the lines $y = 1$, $y = 8$ and $y = 2$. Use this to solve the equations:

(a) $(x+2)^3 = 0$ **(b)** $(x+2)^3 = 1$ **(c)** $(x+2)^3 = 8$ **(d)** $(x+2)^3 = 2$

$x+2 = \sqrt[3]{2}$

$x = \sqrt[3]{2} - 2 \approx -0.74$



2 Solve the following equations algebraically:

(a) $(x+2)^3 = 0$ **(b)** $(x+2)^3 = 1$ **(c)** $(x+2)^3 = 8$

a) $x = -2$

b) $x+2 = 1 \iff x = -1$

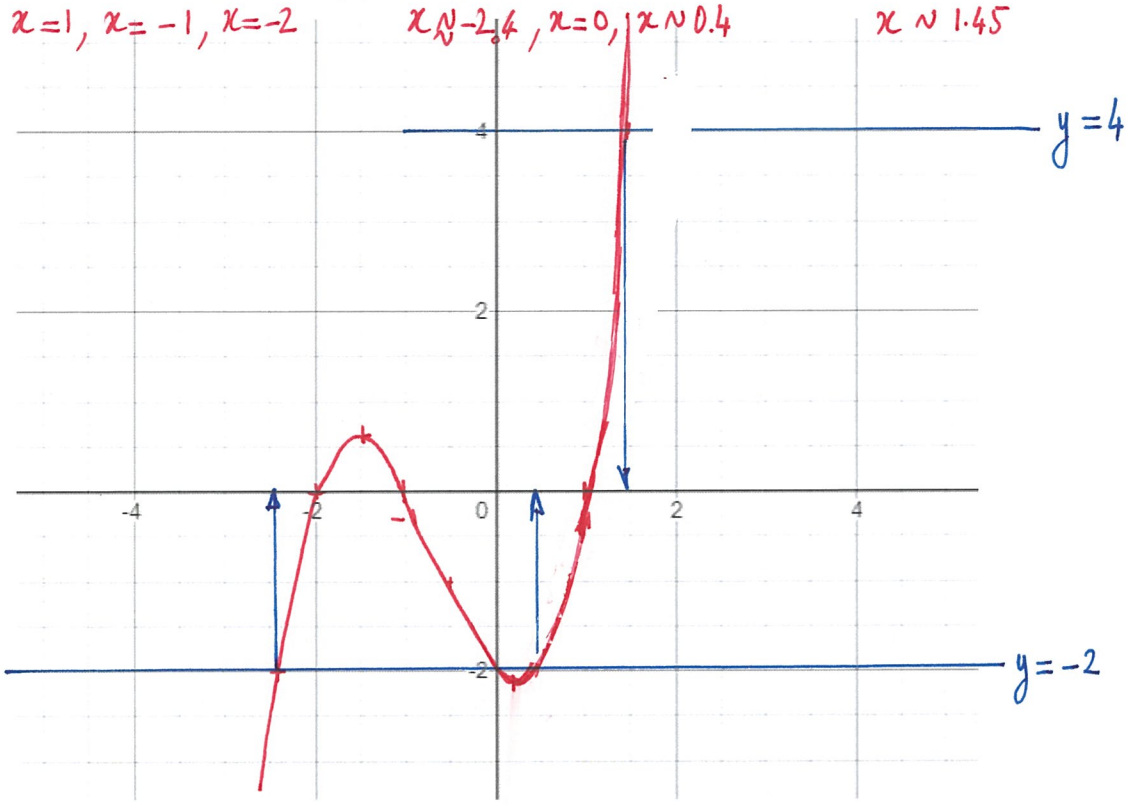
c) $(x+2)^3 = 8 \iff x+2 = \sqrt[3]{8} = 2 \iff x = 0$

CUBIC POLYNOMIALS

3 Draw the graph of $y = (x-1)(x+1)(x+2)$. On your graph, draw the lines $y = -2$ and $y = 4$. Use this to solve the equations:

(a) $(x-1)(x+1)(x+2) = 0$ **(b)** $(x-1)(x+1)(x+2) = -2$ **(c)** $(x-1)(x+1)(x+2) = 4$

$x = 1, x = -1, x = -2$ $x \approx -2.4, x = 0, x \approx 0.4$ $x \approx 1.45$



5 The line $y = c$ is drawn on the graph in question **3**. For what values of c will the equation $(x-1)(x+1)(x+2) = c$ have three distinct roots? Give your answer to one decimal place.

$y = c$ crosses the graph at 3 points when c is between the minimum and the maximum, which are

$c_{min} \simeq -2.1$

$c_{max} \simeq 0.6$

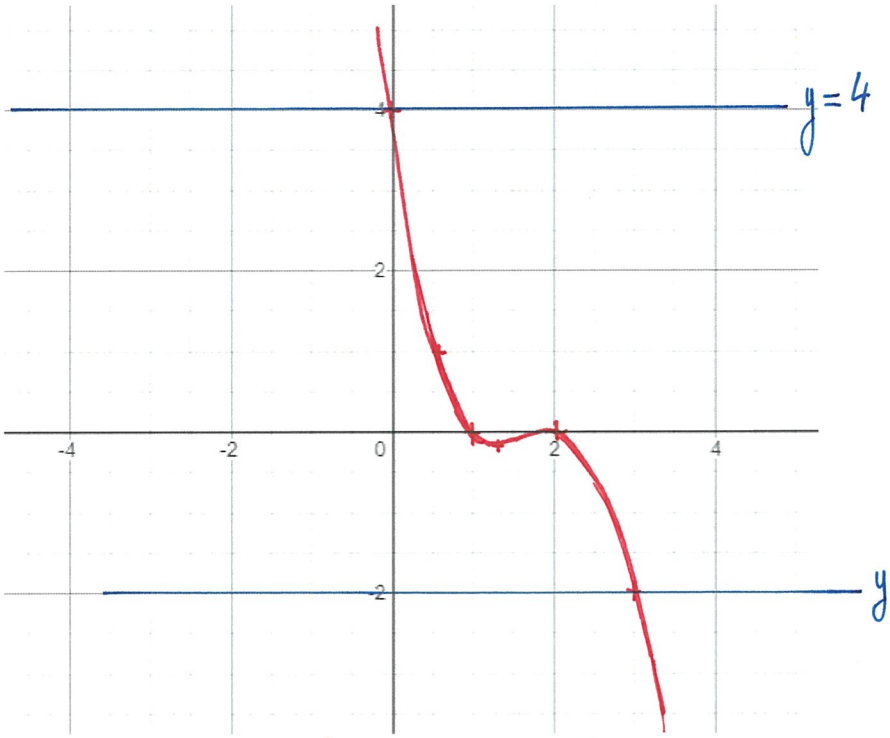
CUBIC POLYNOMIALS

6 Draw the graph of $y=(x-2)^2(1-x)$. By drawing appropriate lines on your graph, use this to solve the following:

(a) $(x-2)^2(1-x)=0$ **(b)** $(x-2)^2(1-x)=4$ **(c)** $(x-2)^2(1-x)=-2$

(d) For what values of c will the equation $(x-2)^2(1-x)=c$ have three distinct roots?

(e) What is the coefficient of x^3 when $(x-2)^2(1-x)$ is expanded?



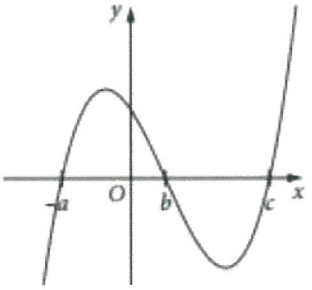
a) $x=1$ and $x=2$ b) $x=0$ c) $x=3$

d) The equation will have 3 distinct roots when $-0.15<c<0$ approx

e) it's (-1)

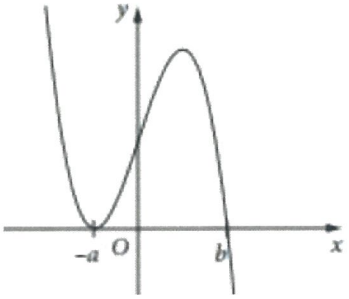
7 For $a, b, c > 0$, the equation of the following graph is best represented by:

A $y=(x-a)(x-b)(x-c)$
B $y=(x-a)(x+b)(x+c)$
C $y=(x-a)(x-b)(x+c)$
D $y=(x+a)(x-b)(x-c)$



8 For $a, b > 0$, the equation of the following graph is best represented by:

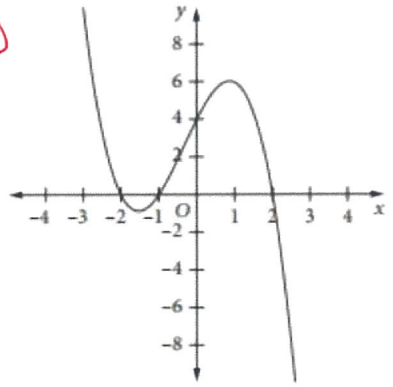
A $y=(x+a)(x-b)^2$
B $y=(x+a)^2(b-x)$
C $y=(a-x)(x+b)^2$
D $y=(x-a)^2(x+b)$



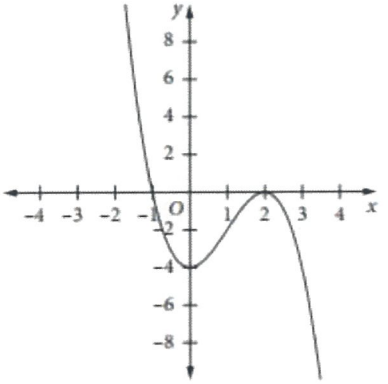
CUBIC POLYNOMIALS

9 If $y = (x + 2)(x + 1)(2 - x)$, which of the following is the graph of this function?

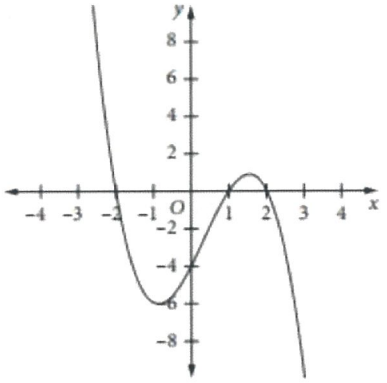
A



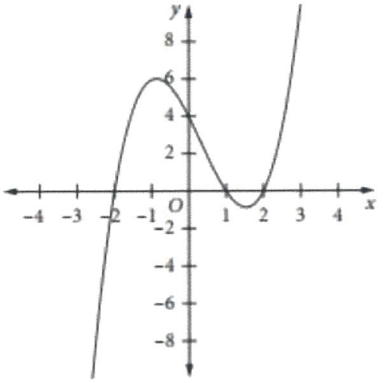
B



C



D



10 Find the equation of the cubic function that cuts the x-axis at $x = -1, 2, 3$ and has a y-intercept of 6.

$y = k(x+1)(x-2)(x-3)$

when $x = 0$, $y = 6 = k \times 1 \times (-2) \times (-3)$ $\Rightarrow k = 1$

$y = (x+1)(x-2)(x-3)$

11 Find the values of a and b if the curve $y = x(x - a)(x + b) + 4$ passes through the points $(1, 0)$ and $(-2, 12)$.

$0 = 1(1-a)(1+b)+4$ $\Rightarrow (1-a)(1+b) = -4 \Leftrightarrow 5 - a + b - ab = 0$ ①

$12 = -2(-2-a)(-2+b)+4$ $\Rightarrow -12 - 8 - 4a + 4b + 2ab + 4 = 0$

$\Leftrightarrow -4a + 4b + 2ab - 16 = 0$

$\Leftrightarrow -4(a-b) + 2ab - 16 = 0$ ②

So $-4 \times$ ① $\Rightarrow 4a - 4b + 4ab - 20 = 0$ adding these 2, we obtain:

$6ab = 36$ $\Rightarrow ab = 6$ $a = 6/b$ giving $-\frac{6}{b} + b - 6 + 5 = 0$

$\Leftrightarrow -\frac{6}{b} + b - 1 = 0$

$-6 + b^2 - b = 0$

$\Delta = 1 - 4 \times (-6) = 25$

$\Delta = 5^2$

$\begin{cases} b = \frac{1-5}{2} = -2 \\ a = -3 \end{cases}$ or $\begin{cases} b = \frac{1+5}{2} = 3 \\ a = -2 \end{cases}$