USING DERIVATIVES IN MOTION IN A STRAIGHT LINE

1 A particle is moving in a straight line so that its displacement x metres is given by $x = \frac{t^3}{2} - 3t^2 + 5$.

(a) Find an expression for its velocity. (b) Find an expression for its acceleration.
(c) When is the velocity zero? (d) Find the displacement, velocity and acceleration after 4 seconds.

a) $x(t) = \frac{t^3}{2} - 3t^2 + 5$ $\quad \dot{x}(t) = \frac{3t^2}{2} - 6t$

b) $\ddot{x}(t) = 3t - 6$

c) $\dot{x}(t) = 0$ when $\frac{3t^2}{2} - 6t = 0$

$\Leftrightarrow \frac{3t^2}{2} = 6t$ so either $t = 0$

or $\frac{3t}{2} = 6 \Leftrightarrow t = \frac{12}{3} = 4$

d) At $t = 4$ $\quad x(4) = \frac{4^3}{2} - 3 \times 4^2 + 5 = -11$

$\dot{x}(4) = \frac{3 \times 4^2}{2} - 6 \times 4 = 0$

$\ddot{x}(4) = 3 \times 4 - 6 = 6$

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3 The displacement x metres at time t seconds, $t \geq 0$, of a particle moving in a straight line is given by $x = 2t^3 - 6t^2 - 30t$.

(a) Find the velocity and acceleration at any time t.
(b) Find the initial velocity and acceleration.
(c) At what time is the velocity zero? What is the acceleration at this time?
(d) During what time interval is the velocity negative?

a) $x(t) = 2t^3 - 6t^2 - 30t$

$\dot{x}(t) = 6t^2 - 12t - 30$

$\ddot{x}(t) = 12t - 12$

b) At $t=0$ $\quad \dot{x}(0) = -30 \quad \ddot{x}(0) = -12$

c) $\dot{x}(t) = 0$ when $6t^2 - 12t - 30 = 0$

$\iff t^2 - 2t - 5 = 0$

$\Delta = 4 - 4\times(-5) = 24$

$t = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}$ but only $1+\sqrt{6}$ is positive.

At that time: $\ddot{x}(1+\sqrt{6}) = 12(1+\sqrt{6}) - 12 = 12\sqrt{6}$

d) $f(t) = t^2 - 2t - 5$ is a concave-up parabola.
with 2 roots $(1-\sqrt{6})$ and $(1+\sqrt{6})$
So $\dot{x}(t)$ is negative between 0 and $(1+\sqrt{6})$
as the negative values of t are not possible

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4 A particle is projected vertically upwards from the ground. The equation for its motion is given by $x = 30t - 5t^2$, where x is the displacement in metres above the ground and t is in seconds.

(a) Graph the displacement function. (b) Find the velocity as a function of time.

(c) What is the initial velocity of the particle?

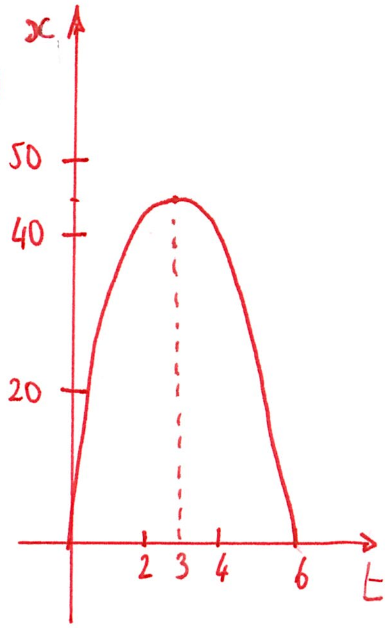
(d) When does the particle reach its greatest height and how high above the ground is it then?

(e) How long will it take before the particle returns to the ground?

(f) What is the particle's speed when it hits the ground?

(g) Find the expression for the acceleration of the particle.

a)



b) $\dot{x}(t) = 30 - 10t$

c) At $t=0$ $\quad \dot{x}(0) = 30$

d) it reaches its greatest height when $\dot{x}(t) = 0$, i.e. when $30 - 10t = 0$

i.e. $t = 3$

At $t=3$ $\quad x(3) = 30 \times 3 - 5 \times 3^2 = 45$

e) $x(t) = 0 \iff 30t - 5t^2 = 0$

$\iff 5t(6-t) = 0$ So either $t=0$ or $t=6$

So it will take another 3s to return to the ground.

f) At $t=6$ $\quad \dot{x}(6) = 30 - 10 \times 6 = -30$

g) $\ddot{x}(t) = -10$

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6 An object moves with a velocity v given by $v = 20 + (2t - 1)e^{-0.5t}$, where t is in hours and v is in km h^{-1}. Calculate:

(a) the velocity after 1 hour

(b) the time taken to reach its maximum velocity.

a) $v(t) = 20 + (2t-1)\,e^{-0.5t}$

At $t=1$ $\quad v(1) = 20 + (2\times 1 - 1)\,e^{-0.5\times 1}$

$v(1) = 20 + e^{-0.5} = 20 + \frac{1}{\sqrt{e}} = 20.61$ km h^{-1}

b) $\dot{v} = 2\times e^{-0.5t} + (2t-1)(-0.5)e^{-0.5t}$ (product rule)

$\dot{v} = e^{-0.5t}\left[2 - t + \frac{1}{2}\right]$

$\dot{v} = e^{-0.5t}\left[\frac{5}{2} - t\right]$

so $\dot{v}(t) = 0$ when $t = 5/2 = 2.5$ hours.