USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

Prove the following identities (questions **1** to **21**):

1 $\dfrac{\sin A+\cos A\tan B}{\cos A-\sin A\tan B}=\tan(A+B)$

2 $\dfrac{\sin 2\theta\cos\theta-\cos 2\theta\sin\theta}{\cos 2\theta\cos\theta+\sin 2\theta\sin\theta}=\tan\theta$

3 $\dfrac{\tan A-\tan B}{\tan A+\tan B}=\dfrac{\sin(A-B)}{\sin(A+B)}$

4 $\sin(\theta+\alpha)\sin(\theta-\alpha)=\sin^2\theta-\sin^2\alpha$

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Prove the following identities (questions **1** to **21**):

9 $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=2\sec 2\theta$

10 $\dfrac{1-\cos x}{\sin x}=\tan\dfrac{x}{2}$

11 $\dfrac{\sin A+\sin(90°-A)+1}{\sin A-\sin(90°-A)+1}=\cot\dfrac{A}{2}$

12 $\dfrac{\sin x+1-\cos x}{\sin x-1+\cos x}=\dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

14 $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \cos B \sin C \sin A - \cos C \sin A \sin B$
What is the resulting identity if B is replaced by $(90° - C)$?

21 $\dfrac{1-\tan\theta\tan 2\theta}{1+\tan\theta\tan 2\theta} = 4\cos^2\theta - 3$

22 If $\tan A = \dfrac{p}{q}$, express the following in terms of p and q.

(a) $q\sin A\cos A + p\sin^2 A$

(b) $p\sin 2A + q\cos 2A$

23 If A, B and C are the angles of a triangle, prove that $\cos A \cos B - \sin A \sin B + \cos C = 0$.

24 Given that $\sin 18° = \frac{1}{4}\left(\sqrt{5}-1\right)$, find $\cos 36°$ in surd form.

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26 Three points P, Q, R are in a horizontal plane. Angles RPQ and RQP are α and β respectively. If PQ is x units in length, show that the perpendicular distance y from R to PQ is given by $y = \dfrac{x \tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$.

29 If $\tan\theta = \frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of: **(a)** $\sin\theta$ **(b)** $\cos\theta$ **(c)** $\cos 2\theta$

31 If $\operatorname{cosec}\alpha = -\frac{17}{8}$ and $\pi < \alpha < \frac{3\pi}{2}$, find the value of: **(a)** $\cot\alpha$ **(b)** $\tan 2\alpha$

37 **(a)** By writing expansions for $\sin(A+B)$ and $\sin(A-B)$, find a simplified expression for $\sin(A+B)+\sin(A-B)$.

(b) By writing $\theta = A+B$ and $\phi = A-B$, find an expression for $\sin\theta + \sin\phi$ as the product of two trigonometric functions.

38 If $\sec\theta - \tan\theta = \frac{3}{5}$, show that $\sin\theta = \frac{8}{17}$. (Hint: Use t formulae.)

39 If $4\tan(\alpha-\beta)=3\tan\alpha$, prove that $\tan\beta=\dfrac{\sin 2\alpha}{7+\cos 2\alpha}$.

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40 Use the factors of $x^3 - y^3$ to show that $\cos^6\theta - \sin^6\theta = \left(1 - \frac{1}{4}\sin^2 2\theta\right)\cos 2\theta$.

41 If $\tan\theta = t$, express $\sin 2\theta$ and $\cos 2\theta$ in terms of t. Find the values of t for which $(k+1)\sin 2\theta + (k-1)\cos 2\theta = k+1$.

44 If $\tan\alpha = k\tan\beta$, show that $(k-1)\sin(\alpha+\beta) = (k+1)\sin(\alpha-\beta)$.