FACTORISING QUADRATIC TRINOMIALS

To factorise quadratic trinomials, you must remember how to expand binomial products and then work backwards. We know the following:

$(x+m)(x+n) = x^2 + (m+n)x + mn$

$(x-m)(x-n) = x^2 - (m+n)x + mn$

$(x+m)(x-n) = x^2 + (m-n)x - mn$

For example, to factorise $x^2 + 5x + 6$ you must write it in the form $(x+m)(x+n)$ where $m+n=5$ and $mn=6$. This means you must find two numbers whose sum is 5 and whose product is 6.

Example 7

Factorise:

(a) x^2+5x+6 **(b)** $x^2-7x+10$ **(c)** x^2+x-12 **(d)** x^2-6x+9 **(e)** $x^2-5x-24$

Solution

(a) x^2+5x+6

Write: $x^2+5x+6=(x+m)(x+n)$

Look for numbers m and n whose sum is 5 and whose product is 6.

List possible factors of 6 and check the sum: $6\times1=6$ $\quad 6+1=7$

$3\times2=6$ $\quad 3+2=5$

Hence: $x^2+5x+6=(x+3)(x+2)$

The information could also be set out using the cross method: $x \times 6 \quad 3$; $x \times 1 \quad 2$

The correct pair will give $5x$ when multiplied across.

(b) $x^2-7x+10$

Write: $x^2-7x+10=(x+m)(x+n)$

Look for numbers m and n whose sum is -7 and whose product is 10.

Since the sum is negative and the product is positive, both the numbers are negative.

List possible factors of 10 and check the sum: $-10\times(-1)=10$ $\quad -10+(-1)=-11$

$-5\times(-2)=10$ $\quad -5+(-2)=-7$

Hence: $x^2-7x+10=(x-5)(x-2)$

(c) x^2+x-12

Write: $x^2+x-12=(x+m)(x+n)$

Look for numbers m and n whose sum is 1 and whose product is -12.

Since the sum is positive and the product is negative, the numbers have different signs and the larger number is positive.

List possible factors of -12 and check the sum: $12\times(-1)=-12$ $\quad 12+(-1)=11$

$6\times(-2)=-12$ $\quad 6+(-2)=4$

$4\times(-3)=-12$ $\quad 4+(-3)=1$

Hence: $x^2+x-12=(x+4)(x-3)$

Using the cross method: $x \times 12 \quad 6 \quad 4$; $x \times -1 \quad -2 \quad -3$

The correct pair will give x when multiplied across.

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(d) $x^2 - 6x + 9$

Write: $x^2 - 6x + 9 = (x + m)(x + n)$

Look for numbers m and n whose sum is -6 and whose product is 9.

Since the sum is negative and the product is positive, both the numbers are negative.

List possible factors of 9 and check the sum: $-9 \times (-1) = 9$ $\quad -9 + (-1) = -10$

$-3 \times (-3) = 9$ $\quad -3 + (-3) = -6$

Hence: $x^2 - 6x + 9 = (x - 3)(x - 3) = (x - 3)^2$

(e) $x^2 - 5x - 24$

Write: $x^2 - 5x - 24 = (x + m)(x + n)$

Look for numbers m and n whose sum is -5 and whose product is -24.

Since the sum is negative and the product is negative, the numbers have different signs and the smaller number is positive.

List possible factors of -24 and check the sum: $-24 \times 1 = -24$ $\quad -24 + 1 = -23$

$-12 \times 2 = -24$ $\quad -12 + 2 = -10$

$-6 \times 4 = -24$ $\quad -6 + 4 = -2$

$-8 \times 3 = -24$ $\quad -8 + 3 = -5$

Hence: $x^2 - 5x - 24 = (x - 8)(x + 3)$

x $\times$ ~~-24~~ ~~-12~~ ~~-6~~ -8

x ~~1~~ ~~2~~ ~~4~~ 3

The correct pair will give $-5x$ when multiplied across.

With practice, you will be able to write the factors simply by looking at the sum and product.