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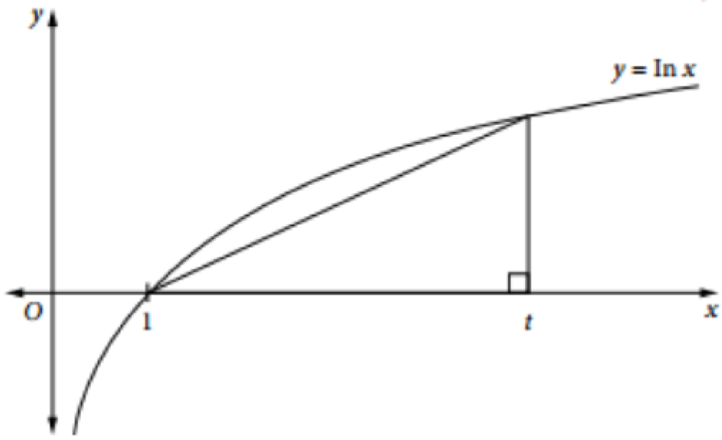
5 Find the volume of the solid generated when the part of the curve $y = \frac{1}{\sqrt{4 - x^2}}$ between $x = -1$ and $x = 1$ is rotated about the x-axis.

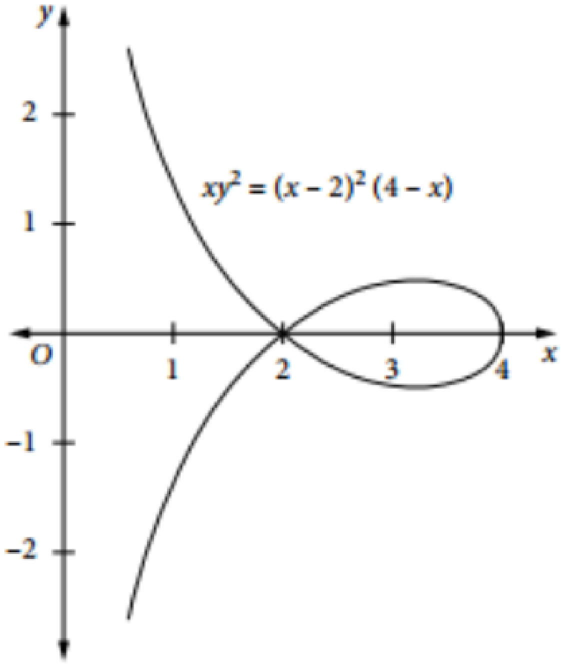
7 The diagram shows the graph of $y = \ln x$.

(a) Find $\int_1^t \ln x \, dx$.

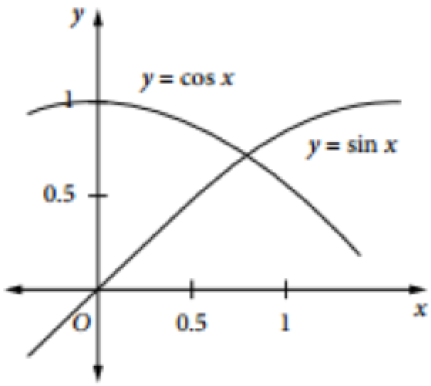
(b) By comparing relevant areas in the diagram, or otherwise, show that $\ln t > 2\left(\frac{t-1}{t+1}\right)$, for $t > 1$.



10 The diagram shows the graph of $xy^2 = (x - 2)^2(4 - x)$. Find the volume of the solid formed by rotating the loop in the graph of $xy^2 = (x - 2)^2(4 - x)$ about the x-axis.



11 **(a)** The region bounded by the curves $y = \sin x$, $y = \cos x$ and the y-axis is rotated about the x-axis. Calculate the volume of the solid of revolution formed.



12 When the circle $x^2 + y^2 = a^2$ is rotated about the line $x = b$ $(b > a)$ to generate a torus, the volume of the solid of revolution formed is given by $V = 4\pi\int_{-a}^{a}(b - x)\sqrt{a^2 - x^2}\,dx$. Calculate this volume.

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14 Find the particular solution of $\frac{dy}{dx} = x^2 \sin x + 2x \cos x - 2 \sin x$, given that $y = 6$ when $x = 0$.

15 Find the particular solution of $\frac{dy}{dx} = e^{-x} \sin x$, given that $y = -\frac{1}{2}$ when $x = 0$.

19 **(a)** The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is rotated about the x-axis. Show that the volume of the solid generated is $\frac{1}{3}4\pi ab^3$ cubic units.

(b) When the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is rotated about the line $x=a$, the volume of the solid of revolution is given by $V=2\pi\int_{-a}^{a}2y(a-x)dx$. Calculate the volume of this solid of revolution.

22 **(a)** Let $I_n = \int_1^2 (\log_e x)^n \, dx$ where n is a positive integer. Show that $I_n = 2(\log_e 2)^n - nI_{n-1}$.

(b) Let $f(x) = (\log_e x)^n$ where integer $n \ge 2$. By considering the first and second derivatives, show that $f(x)$ is increasing and concave up over the domain $1 < x \le 2$. Sketch the curve $y = (\log_e x)^n$ (where $n \ge 2$) over the domain $1 \le x \le 2$.

(c) By considering the area of a triangle that acts as an upper bound, show that: $nI_{n-1} > \frac{3}{2}(\log_e 2)^n$.

(d) Hence show that $\frac{2}{3} < \log_e 2 < 2$.

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26 The base of a solid is formed by the area bounded by $y = \sin x$ and $y = -\sin x$ for $0 < x < \frac{\pi}{2}$ and the line $x = \frac{\pi}{2}$. Vertical cross-sections of the solid taken parallel to the y-axis are isosceles triangles with equal sides of length 1 unit, as shown. The volume of this solid is given by $V = \int_0^{\frac{\pi}{2}} y\sqrt{1-y^2}\,dx$ where $y = \sin x$. Calculate the exact volume of this solid.