INTEGRALS RESULTING IN LOGARITHMIC FUNCTIONS

$\frac{d}{dx}(ln\,x) = \frac{1}{x}$ therefore $\int \frac{1}{x}dx = ln|x| + C$

$\frac{d}{dx}[ln(ax+b)] = \frac{a}{ax+b}$ therefore $\int \frac{1}{ax+b}dx = \frac{1}{a}ln|ax+b| + C$

$\frac{d}{dx}[ln(f(x))] = \frac{f'(x)}{f(x)}$ therefore $\int \frac{f'(x)}{f(x)}dx = ln|f(x)| + C$

Example 10

Find the indefinite integral of the following:

(a) $\frac{2}{2x-3}$ **(b)** $\frac{x}{x^2+4}$ **(c)** $\frac{4x-6}{x^2-3x}$ **(d)** $\frac{e^x}{1+e^x}$

Solution

(a) $\int \frac{2}{2x-3}dx = \int \frac{f'(x)}{f(x)}dx$ where $f(x) = 2x-3$ and $f'(x) = 2$

$\int \frac{2}{2x-3}dx = \log_e|2x-3| + C$

(b) $\int \frac{x}{x^2+4}dx = \frac{1}{2}\int \frac{2x}{x^2+4}dx$

$= \frac{1}{2}\int \frac{f'(x)}{f(x)}dx$ where $f(x) = x^2+4$ and $f'(x) = 2x$

$= \frac{1}{2}\log_e|x^2+4| + C$

(c) $\int \frac{4x-6}{x^2-3x}dx = 2\int \frac{2x-3}{x^2-3x}dx$

$= 2\int \frac{f'(x)}{f(x)}dx$ where $f(x) = x^2-3x$ and $f'(x) = 2x-3$

$= 2\log_e|x^2-3x| + C$

(d) $\int \frac{e^x}{1+e^x}dx = \int \frac{f'(x)}{f(x)}dx$ where $f(x) = 1+e^x$ and $f'(x) = e^x$

$= \log_e|1+e^x| + C$

Example 11

Given $\frac{dy}{dx} = \frac{1}{x}$ and $y = 0$ when $x = 0.5$, express y in terms of x.

Solution

$$\frac{dy}{dx} = \frac{1}{x}$$

$$y = \int \frac{1}{x}\,dx$$

$$y = \log_e |x| + C$$

Where $x = 0.5$, $y = 0$, so: $0 = \log_e \frac{1}{2} + C$

$$0 = -\log_e 2 + C$$

$$C = \log_e 2$$

$$\therefore\ y = \log_e |x| + \log_e 2 \quad \text{or} \quad y = \log_e |2x|$$

Example 12

The gradient of a curve at any point is $\frac{4x}{x^2+1}$ and the curve passes through the point $(0,0)$. Find the equation of the curve.

Solution

$$f'(x) = \frac{4x}{x^2+1}$$

$$\therefore\ f(x) = \int \frac{4x}{x^2+1}\,dx$$

$$= 2\int \frac{2x}{x^2+1}\,dx$$

$$= 2\log_e (x^2+1) + C$$

As $f(0) = 0$: $0 = 2\log_e 1 + C$

$$C = 0$$

$$\therefore\ f(x) = 2\log_e (x^2+1)$$