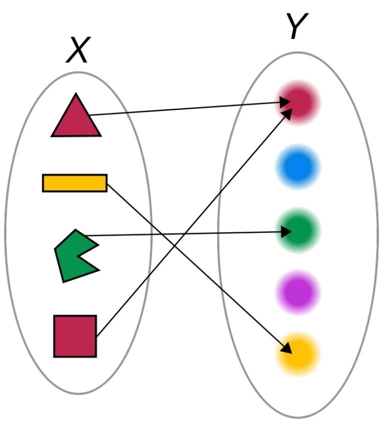
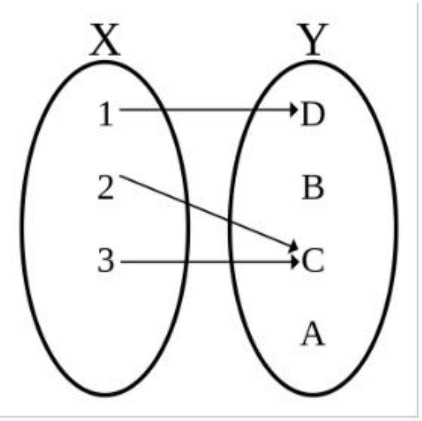
FUNCTIONS

Functions do not have to be a mathematical relationship, they can just be diagrams. Examples



A function that associates any of the four colored shapes to its color.



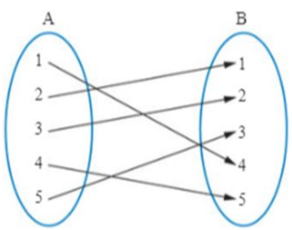
A function that associates to every student its grade

In the second example:

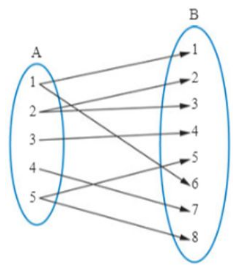
- The function f is defined by the set of pairs {(1,D), (2,C), (3,C)}.
- X={1,2,3} is called the domain of the function
- Y={A,B,C,D} is called the codomain of the function
- D is the image of 1 under f, C is the image of 2 and 3 under f
- The range of a function is the set of all images. In this example, the range is {C,D}.

RELATIONS vs FUNCTIONS

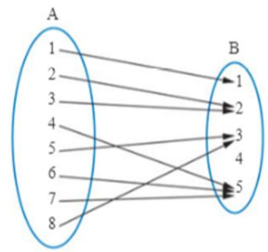
A relation between two sets is a collection of ordered pairs containing one object from each set. If the object is from the first set and the object is from the second set, then the objects are said to be related if the ordered pair is in the relation.



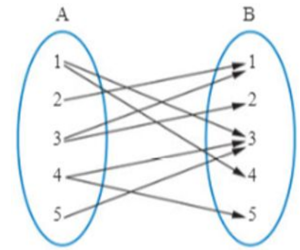
No 1



No 2



No 3



No 4

A function is a type of relation in which no input relates to more than one output.

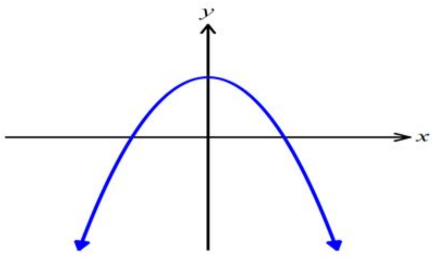
In the diagrams above, only No 1 and No 3 are functions.

FUNCTIONS - THE VERTICAL LINE TEST

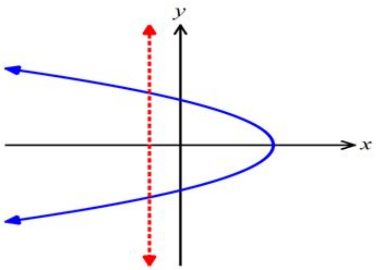
The vertical line test determines whether a graph represents a function. If any vertical line cuts the graph more than once, then it is a relation (not a function).

This is because that x-value has for image at least two different y-values.

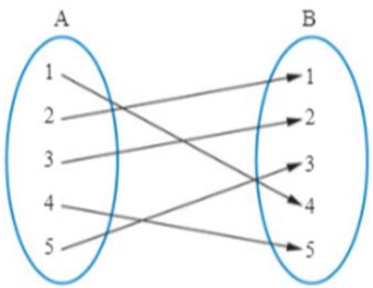
Function



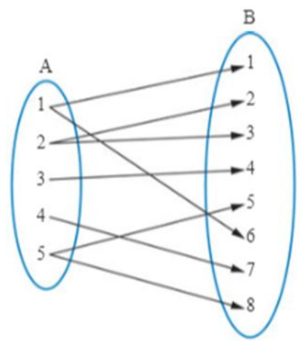
Not a Function



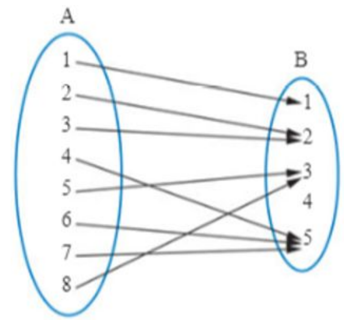
FURTHER CLASSIFICATION OF RELATIONS



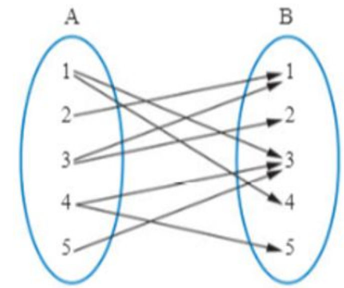
“one-to-one”



“one-to-many”



“many-to-one”



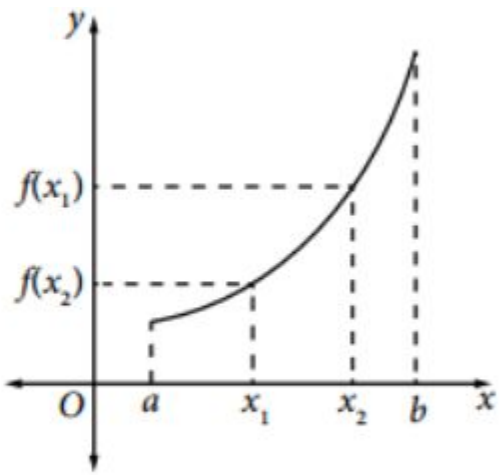
“many-to-many”

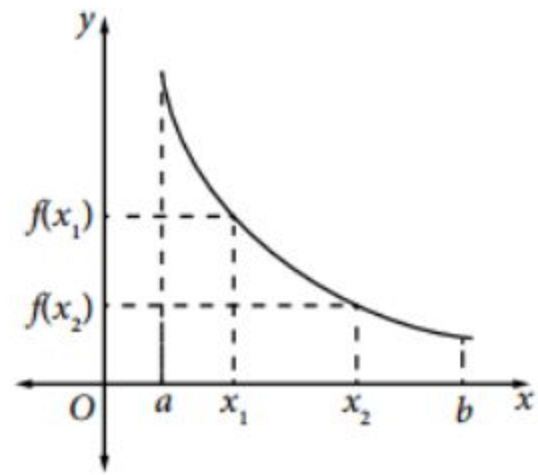
A function is a type of relation in which no input relates to more than one output.

In the diagrams above, <u>only No 1 and No 3 are functions</u>.

INCREASING and DECREASING FUNCTIONS

A function defined on the interval $a \leq x \leq b$ is said to be **strictly monotonic increasing** when, for all x_1 and x_2 in the domain, if $x_2 > x_1$, then $f(x_2) > f(x_1)$. This means that the curve always slopes upwards to the right, as shown in the left diagram below.





Similarly, a function is said to be **strictly monotonic decreasing** when, for all x_1 and x_2 in the domain, if $x_2 > x_1$, then $f(x_2) < f(x_1)$. This means that the curve always slopes upwards to the left, as shown in the right diagram below.

INVERSE FUNCTIONS

An inverse function is a function which reverses, or undoes, another function.

The inverse function is written as $f^{-1}(x)$. The notation $f^{-1}(x)$ is read as "the inverse function of x".

Examples:

if $f(x) = x+2$, then $f^{-1}(x) = x - 2$

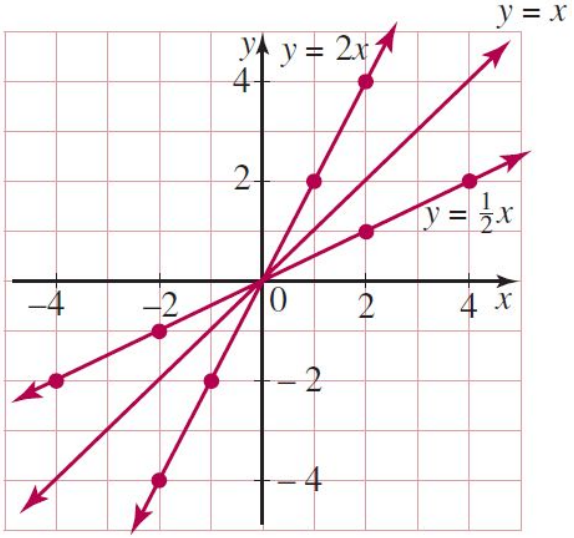
if $f(x) = 2x$, then $f^{-1}(x) = x/2$

If $f(x) = x^3$, then $f^{-1}(x) = \sqrt[3]{x}$

if $f(x) = e^x$, then $f^{-1}(x) = log_e(x)$ [also noted $ln(x)$]

GRAPHING INVERSE RELATIONS AND FUNCTIONS

The geometrical effect of swapping the x- and y-values in each ordered pair of a relation is to produce a reflection of the graph of the relation in the line $y = x$



For example, $y = 2x$ and $y = x/2$ are inverse functions. Each graph is a reflection of the other in the line $y = x$.

THE EQUATION OF THE INVERSE

To find the equation of the inverse of a function (or any relation):

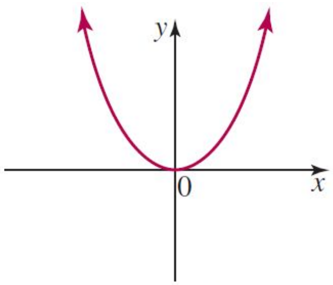
- interchange the variables x and y in the original equation.
- make y the subject of the new equation

For example, the inverse of the function $y = 2x + 1$ is $x = 2y + 1$, which becomes $y = (x - 1)/2$ after rearranging.

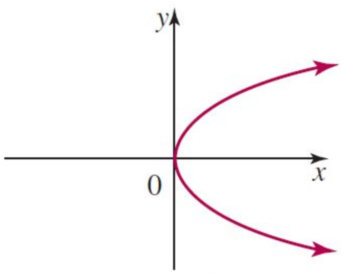
EXISTENCE OF INVERSE FUNCTIONS

THE HORIZONTAL LINE TEST

It does not necessarily follow that the inverse of a function is itself a function. For example, consider the function $y = x^2$ and its inverse $x = y^2$



$y = x^2$



$x = y^2$

The inverse of a function (or any relation) is a function if no horizontal line can be drawn to cut the graph of the original function (or relation) more than once.

A function f will have an inverse function if and only if f is a one-to-one function.