INTRODUCTION TO LIMITS

Definition: a **limit** is the value that a function approaches as its input approaches some value. For a function f which approaches the value L when x tends towards c , this is written as: $\lim_{x\to c} f(x) = L$

and read as "*the limit of the function f when x tends towards c is equal to L*"

$\lim_{x\to 3} 3x = 9$ as $3 \times 3 = 9$

$\lim_{x\to(-1)} (x^2 + 4x) = -3$ as $(-1)^2 + 4 \times (-1) = 1 - 4 = -3$

$$\lim_{x\to 3} \left(\frac{x^2 - 5}{x + 2}\right) = \frac{3^2 - 5}{3 + 2} = \frac{4}{5}$$

From the three examples above, we see it is easy to calculate a limit when the function is continuous; however it is harder for discontinuous functions at their point of discontinuity, as per the examples below:

$\lim_{x\to 0} \left(x + \frac{5}{x}\right) = \infty$ as $\lim_{x\to 0} \frac{1}{x} = \infty$

$$\lim_{x\to 1} \left(\frac{x^3 - 1}{x - 1}\right) = \lim_{x\to 1} \left(\frac{(x - 1)(x^2 + x + 1)}{x - 1}\right) = \lim_{x\to 1} (x^2 + x + 1) = 3$$

$$\lim_{x\to 1^+} \left(\frac{x + 1}{x^2 - 1}\right) = \lim_{x\to 1^+} \left(\frac{x + 1}{(x + 1)(x - 1)}\right) = \lim_{x\to 1^+} \left(\frac{1}{x - 1}\right) = +\infty$$

whereas $\lim_{x\to 1^-} \left(\frac{x+1}{x^2-1}\right) = \lim_{x\to 1^-} \left(\frac{x+1}{(x+1)(x-1)}\right) = \lim_{x\to 1^-} \left(\frac{1}{x-1}\right) = -\infty$

$$\lim_{x\to 2} \left(\frac{x^2 - 8x + 12}{x^2 + 3x - 10}\right) = \lim_{x\to 2} \left(\frac{(x - 2)(x - 6)}{(x - 2)(x + 5)}\right) = \lim_{x\to 2} \left(\frac{x - 6}{x + 5}\right) = -\frac{4}{7}$$

$$\lim_{x\to -\infty} (x^7 - 8x^5 + 12) = \lim_{x\to -\infty} x^7 = -\infty$$

$\lim_{x\to +\infty} \left(\frac{2x+1}{x}\right) = \lim_{x\to +\infty} \left(2 + \frac{1}{x}\right) = 2$ so the line $y = 2$ is an asymptote to the graph of $f(x) = \frac{2x+1}{x}$

$$\lim_{h\to 0} \left(\frac{2xh + h^2 + 7h}{h}\right) = \lim_{h\to 0} (2x + h + 7) = 2x + 7$$

$$\lim_{h\to 0} \left(\frac{3x^2h + 3xh^2 + h^3}{h}\right) = \lim_{h\to 0} (3x^2 + 3xh + h^2) = 3x^2$$