SAMPLE PROPORTION

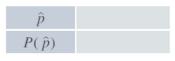**1 a** A student tosses five coins and the results are: H T H T T. What is the sample proportion $\hat{p}$ of heads in this experiment?

b A student selects ten cards from a standard pack with replacement, and records the suit: ♡♣♠◇◇♠♡♠♠◇. What is the sample proportion $\hat{p}$ of spades in this sample?

c A manufacturer takes a sample of 12 items from their recent batch of gizmos, testing each item to see if it passes quality control (P) or not (F). The results were: P P P F F P P P P F P P. What is the proportion $\hat{p}$ of items that pass?

2 A single fair coin is tossed five times, and the number x of heads is recorded.

a Copy and complete the upper table to the right, using the binomial probability formula.

x	0	1	2	3	4	5
$P(X = x)$						

b Copy and complete the lower table to the right to convert part **a** to a table of probabilities of the sample proportions.

$\hat{p}$	
$P(\hat{p})$	

You will need to divide each score x by 5 to obtain the corresponding sample proportion.

c Calculate the mean of the second table.

d How would you interpret this mean?

3 Every Saturday for 20 weeks, a marketer surveyed five people chosen at random in a suburban shopping centre. The first question is, 'Do you live in the suburb?', and the weekly frequency of the number x of 'Yes' answers is given in the upper table.

x	0	1	2	3	4	5
f	1	1	3	2	6	7

a Copy the table to the right, and complete it to show a table in which the two rows are the sample proportions $\hat{p}$ and the relative frequencies f_r.

b Calculate the mean of this second table.

c What is this mean an estimate of?

$\hat{p}$	
f_r	

4 A coin is tossed 10 times, and a student is asked to calculate the probability that more than 75% of the coins will show heads.

a If more than 75% of the coins show heads, how many coins would this be?

b Use the binomial probability formula to determine the probability of obtaining more than 75% heads. (On this occasion the calculation is easily done without a normal approximation.)

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5   A die is thrown 50 times. What is the probability that less than 9% of the time the result will be a six?
Do this:

    a   using an exact binomial calculation, finding the probability of $0,\ 1,\ 2,\ 3$ or 4 sixes

    b   using a normal approximation to the sample proportion, and finding the probability $P(\hat{p} \leq 0.09)$.

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6 Information has been recorded about whether the 32 members of a class buy their lunch regularly at the school canteen:

1.	James	N	2.	Kate	N	3.	Xavier	N	4.	Jimmy	N
5.	Clyde	Y	6.	Bob	N	7.	Liam	N	8.	Agata	N
9.	Irene	N	10.	Aqila	Y	11.	Sonny	Y	12.	Andrea	N
13.	Magarida	N	14.	Terry	N	15.	Iman	N	16.	Lucie	Y
17.	Ping	N	18.	Maddy	Y	19.	Kamal	Y	20.	Xue	Y
21.	Odette	N	22.	Billy	N	23.	Chang	N	24.	Nahla	N
25.	Craig	Y	26.	Jerry	N	27.	Zahra	Y	28.	Jun	N
29.	Nara	Y	30.	Dakarai	Y	31.	Lerato	N	32.	Sahar	Y

In order to generate random samples, each student has been given a unique identifying number.

a Calculate the fraction of students who buy their lunch regularly at the canteen. This is called the *population proportion*.

b Kamal generates the five random numbers 12 15 3 30 17, thus generating the sample of 5 students Andrea, Iman, Xavier, Dakarai, Ping. What proportion of these five students buy their lunch regularly at the canteen? This is called a *sample proportion*.

c Copy the table below. Then use the following sets of five random numbers to generate 10 sample proportions for $n = 5$. Enter your results in the table — the first sample was dealt with in part b and is already included in the tally. (Notice that repetition is allowed — this is sampling with replacement.)

12	15	3	30	17		3	25	17	17	20		27	7	24	26	2		20	9	21	10	16
26	6	11	5	25		29	24	23	27	3		22	11	25	9	8		27	14	22	11	20
9	28	1	17	1		10	32	24	30	13												

$\hat{p}$	0	0.2	0.4	0.6	0.8	1.0
Tally						
Frequency						

d Draw a dot plot for the distribution obtained in part c.

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7   In a local election, 20% of the people voted independent. What is the probability that if 500 people are chosen for a random survey, more than 22% of them voted independent? Use a normal approximation to the sample proportion.

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8 A card is selected from a standard pack and it is then returned. This experiment is repeated 80 times. What is the probability that a hearts card turns up between 20% and 30% of the time, inclusive? Do this:

   a By interpreting it as between 16 to 24 hearts inclusive and using a normal approximation with a continuity correction;

   b Using sample proportion without any correction.

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9 A farmer knows that a certain type of seed is 70% likely to germinate when planted. He plants 300 seeds at the start of the season. Use the normal approximation for the sample proportion to find the probability that:

   a at least 65% will germinate,

   b between 65% and 75% will germinate.

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10 A medication causes a painful reaction in 5% of users.

  a In a group of 100 people, find the probability that:

    i no one reacts,

    ii less than two per cent of the people react.

  b In a larger study into patients' reactions to this medication, 1000 patients are given the medication (to which 5% are known to have a painful reaction). Researchers find that less than 3% of the patients in this study have a reaction.

    i Use the normal approximation to the sample proportion to determine the probability of this happening by chance.

    ii What should the researchers conclude?

12 In this question we investigate the accuracy of a normal approximation to the sample proportion for various sample sizes n.

A coin is tossed repeatedly and the proportion of heads is recorded. The exact theoretical probability of obtaining at most 52% heads is calculated using the binomial distribution, and recorded in the second row of the table below for differing sample sizes.

n	1000	500	100	50	25
exact	0.9026	0.8262	0.6914	0.6641	0.6550
approx					
% error					

a Use a normal approximation for the sample proportion to fill in the second row of the table.

b In each case calculate the percentage error in the approximation for each of these samples. Record your results in the third row of the table.

c Comment on the accuracy of your approximations for various sample sizes n.

13 A manufacturer distributes tins of pineapple under a recognised brand name, and also as a generic supermarket no-name product. In a trial, 50 customers are given a tin of each and later asked to express a preference. The customers in the trial must choose one product as their favourite. Assuming that there is no difference between the two products, the probability of choosing the branded pineapple should be 0.5. It is found that more than 60% of customers prefer the branded pineapple. Calculate the probability of this using the normal approximation for the sample proportion, then comment.

14 Long-term trials have showed that 30% of patients with a certain disease respond to treatment by a company's drug. In further trials, 100 patients chosen at random from those with the disease are given a higher than usual dosage of the drug, and 40% respond positively. What is the probability that 40% or more could respond positively purely by chance?

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17 Two dice are thrown and success is recorded if the sum is at least 9.

  a Find the probability of success.

  b Use the exact binomial distribution to find the probability of at most four successes on 20 throws.

  c Use a normal approximation to the binomial to estimate the probability of at most four successes without continuity correction.

  d Repeat part **c** with continuity correction.

  e Use sample proportion to estimate the probability of at most 20% successes. Do not use continuity correction. Explain why your result agrees with part **c**.

  f Use sample proportion, but this time use an estimate with continuity correction, $P\left(0 - \frac{1}{40} \le \hat{p} \le 0.2 + \frac{1}{40}\right)$. Explain why your result agrees with part **d**.

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z	second decimal place									
	+ .00	+ .01	+ .02	+ .03	+ .04	+ .05	+ .06	+ .07	+ .08	+ .09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Alternatively you can use:

Casio fx100-AU PLUS:	Mode STAT, AC, Shift 1 ("STAT"), 5 ("Distr"), 1 (":P("), z-score
Excel	=NORM.S.DIST(z-score,TRUE)