COMPLEX NUMBERS - CHAPTER REVIEW

1 If $z = 1 + 2i$ and $w = -3 - 4i$, find the following in $x + iy$ form:

(a) $3z + w$ **(b)** z^2 **(c)** $w\overline{w}$ **(d)** $\frac{z}{w}$ **(e)** the square roots of w.

6 **(a)** Evaluate the following, giving answers in both mod–arg form and $x + iy$ form.

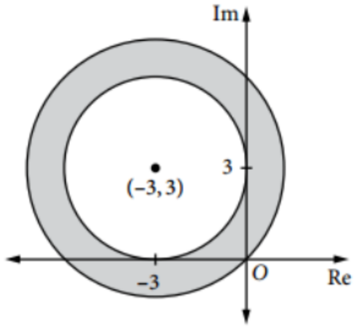
(i) $(\sqrt{3} - i)^3$ **(ii)** $\dfrac{(1-\sqrt{3}i)^2}{(1+i)^3}$

(b) Use your answer to part **(a)(ii)** to show that $\cos\dfrac{7\pi}{12} = \dfrac{\sqrt{2}-\sqrt{6}}{4}$.

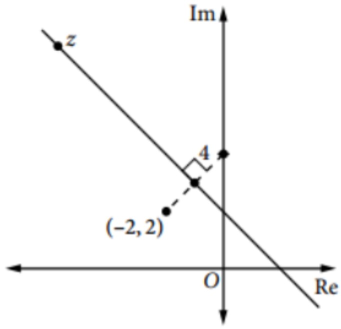
7 If $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ and $w = \cos\left(-\frac{3\pi}{10}\right) + i\sin\left(-\frac{3\pi}{10}\right)$, find $\frac{z^2}{w^5}$ in mod–arg form.

8 Describe each of the following regions of the Argand diagram algebraically.

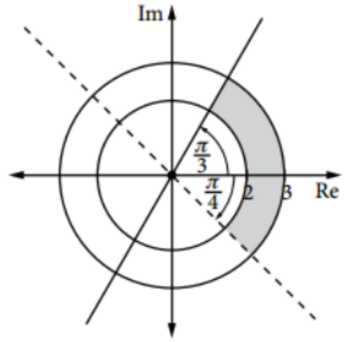
(a)



(b)



(c)



10 Find $\left(1 + i\sqrt{2}\right)^3$

11 If $z = \cos\theta + i\sin\theta$:

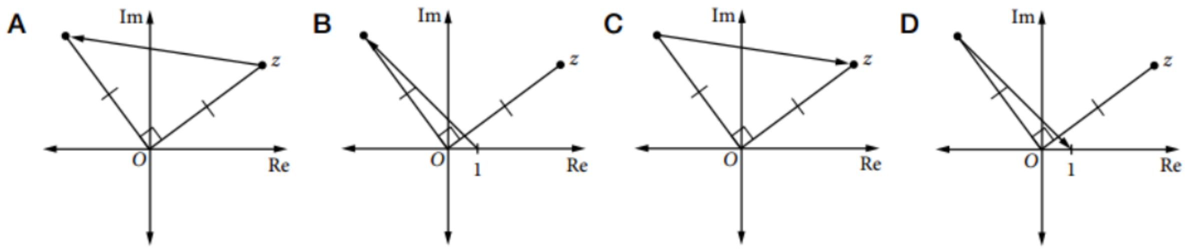
(a) Show that $\arg(z^2 + z^4) = 3\theta$. **(b)** Show that $z^2 + z^4 = 2\cos\theta(\cos 3\theta + i\sin 3\theta)$.

(c) Find the value(s) of θ for which $z^2 + z^4$ is purely imaginary, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

13 **(a)** If w is a root of $z^{12} = i$, show that $-w$ is also a root.

(b) Let z_1 and z_2 be two distinct roots of $z^{12} = i$. Show that $|z_1 + z_2| < 2$.

15 On an Argand diagram, point Z is shown to represent the complex number z. Which diagram below shows the vector that represents $(1 - i)z$?



16 On an Argand diagram, the points A, B, C and D represent the complex numbers α, β, γ and δ respectively.

(a) Describe the point that represents $\frac{1}{2}(\alpha+\gamma)$.

(b) If $\alpha+\gamma=\beta+\delta$, deduce that $ABCD$ is a parallelogram.

18 On an Argand diagram, the points A and C represent the complex numbers $3i$ and $4-5i$ respectively. $ABCD$ is a rhombus.

(a) Find the Cartesian equation of the diagonal BD.

(b) Show that the diagonal BD is also represented by the equation $(1+2i)z+(1-2i)\overline{z}-8=0$.

19 If w is a non-real root of the equation $z^5 = 1$, show that:

(a) $1 + w + w^2 + w^3 + w^4 = 0$ **(b)** $(1 - w)(1 - w^2)(1 - w^3)(1 - w^4) = 5$

(c) $z_1 = w + w^4$ and $z_2 = w^2 + w^3$ are the roots of the quadratic equation $z^2 + z - 1 = 0$.

20 **(a)** Find the cube roots of -8 in mod–arg form.

(b) If w_1 and w_2 are the non-real roots of -8, show that $w_1^{6n} + w_2^{6n} = 2^{6n+1}$ for all integers n.

21 On an Argand diagram, A represents the complex number $z = \cos\theta + i\sin\theta$. B represents wz, where $w = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$. M is the midpoint of OB.

(a) Show that $\overline{AM} = \frac{1}{2}wz - z$.

(b) Show that $\left|\frac{1}{2}wz - z\right| = \sqrt{2-\sqrt{2}}$.

(c) Show that $\arg\left(\frac{1}{2}wz - z\right) = \frac{5\pi}{8} + \theta$.

24 **(a)** Given that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ (see Example **17**, page 16), solve $x^3 - 3\sqrt{3}x^2 - 3x + \sqrt{3} = 0$.

(b) Show that $\tan\dfrac{\pi}{9} - \tan\dfrac{2\pi}{9} + \tan\dfrac{4\pi}{9} = 3\sqrt{3}$.

30 The polynomial $P(x) = ax^3 + bx + c$ has a multiple zero at $x = 2$ and has a remainder of 20 when divided by $x + 2$. Find a, b and c.