INDUCTION - WHEN STEP 2 WORKS, BUT NOT STEP 1

1 Let $S(n)$ be the statement: $n^2 + n$ is an odd integer.

(a) Show that if $S(k)$ is true, then $(k+1)$ is true. **(b)** Is $S(1)$ true? **(c)** Is $S(n)$ true for any n?

(d) If the statement is not true, what change do you need to make to make it true? Prove your new statement.

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2 Given $1^2 + 4^2 + 7^2 + \ldots + (3n-2)^2 = \frac{n}{2}\left(6n^2 - 3n - 1\right)$.

(a) Show that if $S(k)$ is true then $S(k+1)$ is true. **(b)** Is $S(1)$ true? **(c)** Is $S(n)$ true for any n?

INDUCTION - WHEN STEP 2 WORKS, BUT NOT STEP 1

3 It is stated that $n^2 - n + 41$ is prime for $n \geq 1$.

(a) Is this statement true for $n = 1$? **(b)** Is this statement true for $n = 2$?

(c) Is this statement true for $n = 5$?

(d) Is it possible to find a value of n for which this expression does not give a prime number? Justify your answer.

(e) Is the given statement true or false?

INDUCTION - WHEN STEP 2 WORKS, BUT NOT STEP 1

4 Let $S(n)$ be the statement: $n^2 - n$ is an odd integer.

(a) Show that if $S(k)$ is true then $S(k+1)$ is true.

(b) Is $S(1)$ true?

(c) Is $S(n)$ true for any n?