BINOMIAL DISTRIBUTION

1 Find the stated probability for the following binomial distributions. Give your answers correct to four decimal places (d.p.).

(a) $P(X = 4)$ if $X \sim B(12, 0.7)$ (b) $P(X = 6)$ if $X \sim B(20, 0.45)$

(c) $P(X = 2)$ if $X \sim B(8, 0.3)$ (d) $P(X = 30)$ if $X \sim B(50, 0.6)$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} = {}^nC_x \, p^x (1-p)^{n-x}$$

a) $P(X=4) = {}^{12}C_4 \times 0.7^4 \, (1-0.7)^{12-4} = {}^{12}C_4 \times (0.7)^4 \times (0.3)^8 = 0.0078$

b) $P(X=6) = {}^{20}C_6 \times (0.45)^6 \times (1-0.45)^{20-6} = {}^{20}C_6 \times (0.45)^6 \times (0.55)^{14} = 0.0746$

c) $P(X=2) = {}^8C_2 \times (0.3)^2 \times (1-0.3)^6 = {}^8C_2 \times (0.3)^2 \times (0.7)^6 = 0.2965$

d) $P(X=30) = {}^{50}C_{30} \times (0.6)^{30} \times (1-0.6)^{50-30} = {}^{50}C_{30} \times (0.6)^{30} \times (0.4)^{20}$

$= 0.1146$

3 A standard six-sided die is rolled 50 times. Find the probability, correct to five decimal places, of the following outcomes:

(a) exactly 30 even numbers are rolled (b) at least one 6 is rolled.

a) $P(X=30) = {}^{50}C_{30} \times \left(\frac{1}{2}\right)^{30} \times \left(1-\frac{1}{2}\right)^{20} = {}^{50}C_{30} \times (0.5)^{30} \times (0.5)^{20} = 0.04186$

b) P(at least one 6 is rolled) = 1 – P(no 6 is rolled)

$= 1 - \left(\frac{5}{6}\right)^{50}$

$= 0.99989$

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$p = 0.6$

4 Peter, a keen gardener, knows from past experience that only 60% of his tulip bulbs will flower. He plants 20 bulbs. Find the probability, correct to five decimal places, of the following outcomes:

(a) exactly 15 bulbs flower

(b) more than 15 bulbs flower.

a) $P(X=15) = {}^{20}C_{15}\,(0.6)^{15}(1-0.6)^{5} = {}^{20}C_{15}\,(0.6)^{15}(0.4)^{5} = 0.07465$

b) $P(X>15) = P(X=16) + P(X=17) + P(X=18) + P(X=19) + P(X=20)$

$= {}^{20}C_{16}(0.6)^{16}(0.4)^{4} + {}^{20}C_{17}(0.6)^{17}(0.4)^{3} + {}^{20}C_{18}(0.6)^{18}(0.4)^{2} + {}^{20}C_{19}(0.6)^{19}(0.4) + {}^{20}C_{20}(0.6)^{20}$

$= 0.05095$

6 If 100 cards are drawn with replacement from a standard pack of 52 playing cards, find the probability of the following events, correct to four decimal places:

(a) exactly 25 hearts are drawn

(b) exactly 40 cards are clubs

(c) exactly 55 red cards are drawn

(d) exactly 60 cards are black.

a) $p(X=25) = {}^{100}C_{25}\left(\frac{1}{4}\right)^{25}\left(1-\frac{1}{4}\right)^{100-25} = 0.0918$

b) $p(X=40) = {}^{100}C_{40}\left(\frac{1}{4}\right)^{40}\left(\frac{3}{4}\right)^{100-40} = 0.0003626$

c) $p(X=55) = {}^{100}C_{55}\left(\frac{1}{2}\right)^{55}\left(\frac{1}{2}\right)^{100-55} = {}^{100}C_{55}\left(\frac{1}{2}\right)^{100} = 0.04847$

d) $p(X=60) = {}^{100}C_{60}\left(\frac{1}{2}\right)^{60}\left(\frac{1}{2}\right)^{100-60} = {}^{100}C_{60}(0.5)^{100}$

So $p(60\text{ Black}) = 0.01084$

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7 The probability that an archer will shoot a bullseye with any particular arrow is $\frac{4}{5}$. The archer shoots 10 arrows. Which of the following expressions best represents the probability that the archer misses the bullseye with exactly two of the arrows?

A $\binom{10}{2}\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^8$ $\boxed{\text{B}}$ $\binom{10}{8}\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^2$ C $\binom{8}{2}\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^8$ D $\binom{8}{2}\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^2$

8 In Amazonia, 62% of all births are girls. Find the probability, correct to four decimal places, that of eight births exactly half are girls.

$P(X=4) = {}^{8}C_{4} \times (0.62)^4 (1-0.62)^{8-4}$

$= 0.2157$

9 In the town of Sinistraville, 35% of the population is left-handed. Find the probability, correct to four decimal places, that exactly 35 people out of a random sample of 100 will be left-handed.

$P(X=35) = {}^{100}C_{35} (0.35)^{35} (1-0.35)^{65}$

$= 0.0834$

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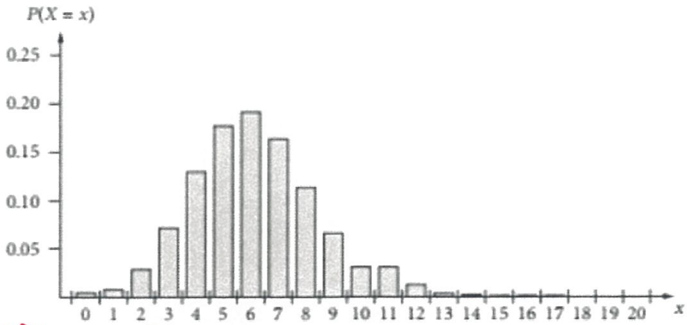
11 For the graph shown, choose the best representation for its binomial distribution.

A $X \sim B(12, 0.5)$ $\boxed{\text{B}}$ $X \sim B(20, 0.3)$

C $X \sim B(12, 0.3)$ D $X \sim B(20, 0.5)$



Graph is skewed so $p \neq 0.5$

There are values for $n > 12$ so $n = 20$

13 On the way to work, Philomena must drive through six sets of traffic lights. The lights are independent of each other and the probability that Philomena must stop at any particular set is 0.7. Find the probability, correct to four decimal places, that Philomena stops at the following sets of lights:

(a) exactly five sets of lights
(b) fewer than two sets of lights
(c) more than five sets of lights
(d) the first three sets of lights
(e) the second and sixth sets of lights only
(f) four sets of lights, including the first two.

a) $P(X=5) = {}^6C_5\,(0.7)^5\,(1-0.7)^1 = 0.3025$

b) $P(X<2) = P(X=0) + P(X=1) = {}^6C_0\,(0.7)^0(0.3)^6 + {}^6C_1\,(0.7)^1(0.3)^5$

$= 0.0109$

c) $P(X>5) = P(X=6) = {}^6C_6\,(0.7)^6 \times (0.3)^0 = 0.1176$

d) Probability $= 0.7^3 = 0.343$

e) Probability $= (0.7)^2 \times (0.3)^4 = 0.00397$

f) Probability $= (0.7)^2 \times \left[{}^4C_2 \times (0.7)^2 \times (0.3)^2\right] = 0.1297$

14 Boxes of matches are supposed to contain 47 matches. Production records indicate that 80% of boxes contain 47 matches. A batch of 20 boxes is sampled. If more than four boxes do not contain exactly 47 matches, production is stopped.

(a) Find the probability, correct to four decimal places, that of the 20 boxes selected, the number of boxes that do not contain 47 matches is the following:

(i) 0 boxes **(ii)** 1 box **(iii)** 2 boxes **(iv)** 3 boxes **(v)** 4 boxes

(b) Find the probability that the number of boxes that do not have 47 matches is no more than four.

(c) Find the probability that production is stopped.

a) i) $P(X=0) = {}^{20}C_0 (0.8)^{20} \times (0.2)^{0} = 0.0115$

ii) $P(X=1) = {}^{20}C_1 (0.2)^1 (0.8)^{19} = 0.0576$

iii) $P(X=2) = {}^{20}C_2 (0.2)^2 (0.8)^{18} = 0.1369$

iv) $P(X=3) = {}^{20}C_3 (0.2)^3 (0.8)^{17} = 0.2054$

v) $P(X=4) = {}^{20}C_4 (0.2)^4 (0.8)^{16} = 0.2182$

b) $P(X \leqslant 4) = \sum_{i=0}^{4} P(X=i) = 0.6296$

c) P(production stopped) = P(more than 4 boxes do not contain 47 matches)

$= 1 - P(X \leqslant 4)$

$= 1 - 0.6296$

$= 0.3704$

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16 A recording company receives a large number of new songs from various artists. On average, only 4% of the new songs become popular hits. One of the producers decides to select a random sample of 35 new songs.

(a) What is the probability that a maximum of three new songs from the sample will become popular hits? State your answer correct to three decimal places.

(b) What is the probability that more than four but less than six songs from the sample will become popular hits? State your answer correct to three decimal places.

(c) Determine the most likely number of songs from this sample that will become popular hits. Explain your answer using appropriate calculations.

(d) The producer decides to select 70 songs at random from the new songs received. If the probability of a new song becoming a popular hit has not changed, what is the probability that exactly one new song will become a popular hit?

(e) What would be the most likely number of new songs from this sample to become popular hits? Explain your answer using appropriate calculations.

probability (one song is popular) $= 0.04$.

a) $P(X=0)+P(X=1)+P(X=2)+P(X=3) = {}^{35}C_0 \times (0.04)^0 (0.96)^{3}$
$+ {}^{35}C_1 (0.04)^1 (0.96)^{34} + {}^{35}C_2 (0.04)^2 (0.96)^{33} + {}^{35}C_3 (0.04)^3 (0.96)$

$= 0.9499$ so 0.950 to 3 d.p.

b) $P(X=5) = {}^{35}C_5 \times (0.04)^5 (0.96)^{30} = 0.00977$

so 0.010 to 3 d.p.

c) The graph is skewed towards the y-axis, i.e

$P(X=0) = {}^{35}C_0 \times (0.04)^0 \times (0.96)^{35} \simeq 0.24$

$P(X=1) = {}^{35}C_1 \times (0.04)^1 \times (0.96)^{34} \simeq 0.35$

$P(X=2) = {}^{35}C_2 \times (0.04)^2 \times (0.96)^{33} \simeq 0.24$

so $P(X=1)$ has the greatest value. 1 song

d) $P(X=1) = {}^{70}C_1 \times (0.04)^1 \times (0.96)^{69} \simeq 0.167$

e) $P(X=0) = {}^{70}C_0 \times (0.04)^0 \times (0.96)^{70} \simeq 0.057$

$P(X=2) = {}^{70}C_2 \times (0.04)^2 \times (0.96)^{68} \simeq 0.24$

$P(X=3) = {}^{70}C_3 \times (0.04)^3 \times (0.96)^{67} \simeq 0.23$

so $P(X=2)$ has the greatest value. 2 songs