USING ARRAYs FOR TWO-STEPs EXPERIMENTS

A table called an array may be used to list the sample space of a two-steps experiment.

Example: in a basket containing a blue, red and green ball, picking a ball at random, put it back, then picking another one.

	Blue	Red	Green
Blue	(Blue,Blue)	(Blue,Red)	(Blue,Green)
Red	(Red,Blue)	(Red,Red)	(Red,Green)
Green	(Green,Blue)	(Green,Red)	(Green,Green)

ARRAY - WITH REPLACEMENT

<u>If replacement is allowed</u>, then outcomes from each selection can be repeated.

Example: two selections are made from digits $\{1, 2, 3\}$

If replacement is allowed, the possible outcomes are:

		1st		
		1	2	3
2nd	1	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

ARRAY - WITHOUT REPLACEMENT

If replacement is NOT allowed, the possible outcomes are:

		1st		
		1	2	3
2nd	1	×	(2, 1)	(3, 1)
	2	(1, 2)	×	(3, 2)
	3	(1, 3)	(2, 3)	×

A fair 6-sided die is rolled twice.

a List the sample space, using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome (1, 5).

d Find:

i $P(\text{double})$ **ii** $P(\text{sum of at least } 10)$ **iii** $P(\text{sum not equal to } 7)$

e Find the probability of a sum of 12, given that the sum is at least 10.

a

Roll 2

		1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Roll 1	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Be sure to place the number from roll 1 in the first position for each outcome.

b 36 outcomes

c $P(1, 5) = \frac{1}{36}$

d **i** $P(\text{double}) = \frac{6}{36}$

ii $P(\text{sum of at least } 10) = \frac{6}{36} = \frac{1}{6}$

iii $P(\text{sum not equal to } 7) = 1 - \frac{6}{36}$

e $P(\text{sum} = 12 \text{ given that sum} \geqslant 10) = \frac{1}{6}$

PRODUCT RULE for INDEPENDENT EVENTS

If A and B are two independent events, then the probability that the event A will occur, followed by event B, is given by:

$$P(AB) = P(A) \times P(B)$$

P(A) is the probability that event A will occur.

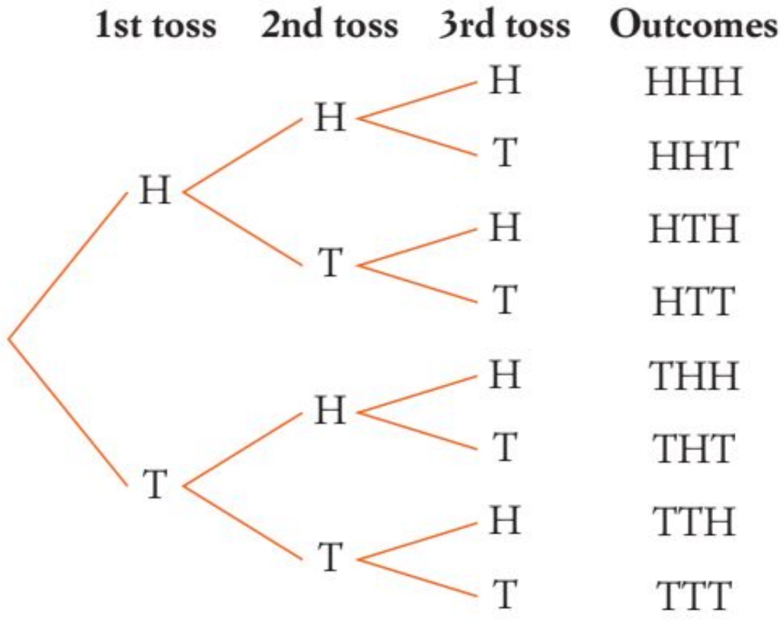
P(B) is the probability that event B will occur.

P(AB) is the probability that events A and B will occur in that order.

TREE DIAGRAMS

A tree diagram is a systematic way of listing all the possible outcomes in a multi-stage event.

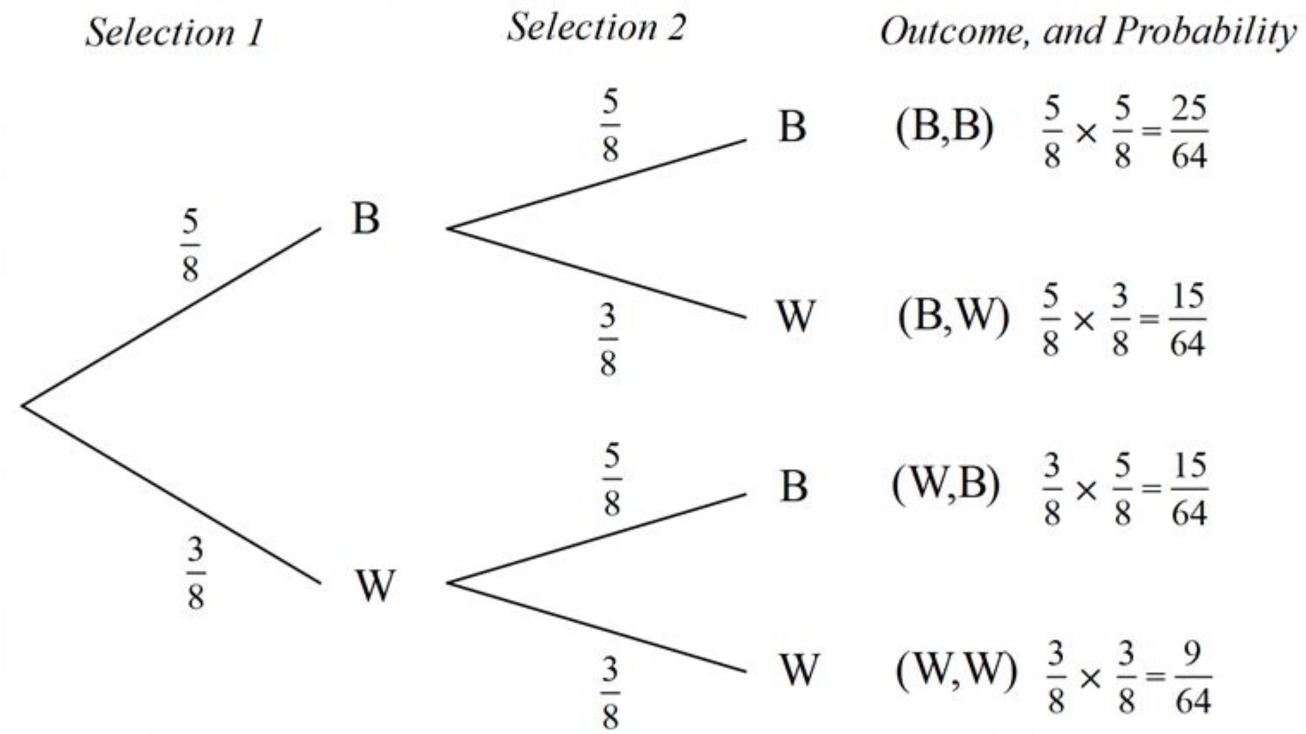
This tree diagram shows the possible outcomes when a coin is tossed three times:



TREE DIAGRAM - WITH REPLACEMENT

A bag contains 8 marbles: 5 blue (B) and 3 white (W).

Two marbles are selected WITH replacement.

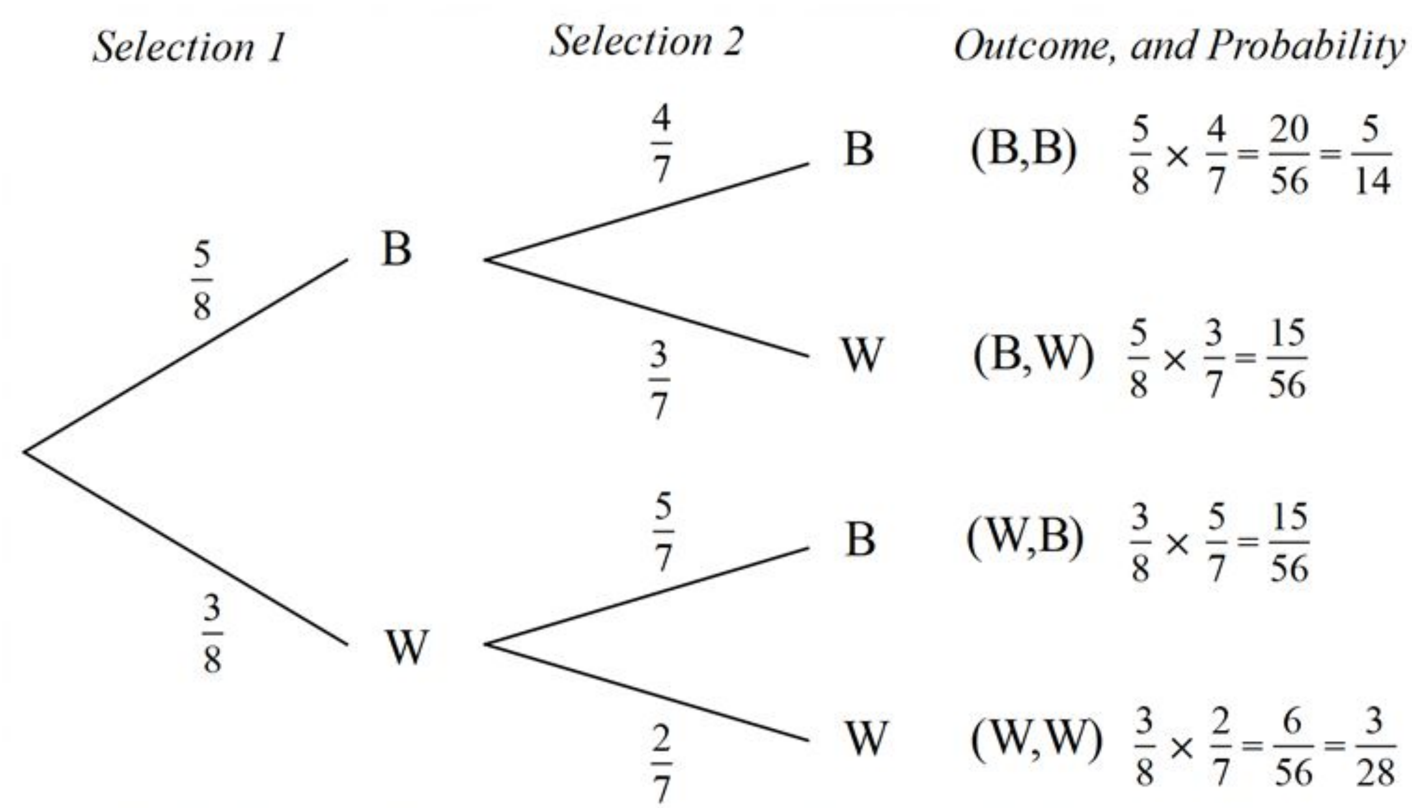


Each outcome for the experiment is obtained by multiplying the branch probabilities.

TREE DIAGRAM - WITHOUT REPLACEMENT

A bag contains 8 marbles: 5 blue (B) and 3 white (W).

Two marbles are selected WITHOUT replacement.



TREE DIAGRAMS

For two-stage events, such as rolling two dice, a table is more convenient than a tree diagram for listing the sample space. example:

Two dice are rolled and their sum is calculated.

a Use a table to list all possible sums.

b What is the probability of rolling a sum of 10?

		2nd die					
	+	**1**	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	**10**
	5	6	7	8	9	**10**	11
	6	7	8	9	**10**	11	12

b There are 36 possible outcomes.

$$P(\text{sum of } 10) = \frac{3}{36}$$
$$= \frac{1}{12}$$