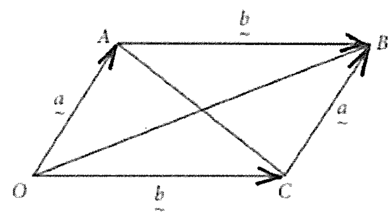
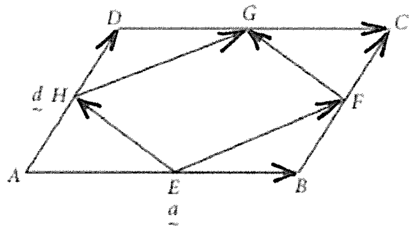
VECTORS IN GEOMETRIC PROOFS

1 Consider the parallelogram $OABC$ where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{b}$. Express $\overrightarrow{OB}$ and $\overrightarrow{CA}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Hence show that the diagonals of a parallelogram meet at right angles if and only if it is a rhombus.



2 Consider the parallelogram $ABCD$, where $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{AD} = \underset{\sim}{d}$. Prove that the midpoints of the sides of a parallelogram join to form a parallelogram.



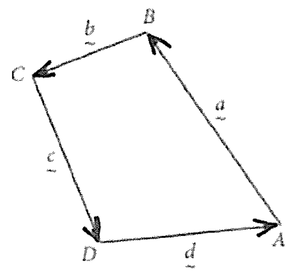
3 Consider the quadrilateral $ABCD$, as shown.

Let $\overrightarrow{AB} = \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{b}$, $\overrightarrow{CD} = \underset{\sim}{c}$ and $\overrightarrow{DA} = \underset{\sim}{d}$.

Which one of the following statements is correct?

A $\underset{\sim}{a} - \underset{\sim}{c} = \underset{\sim}{b} - \underset{\sim}{d}$ **B** $\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{c} + \underset{\sim}{d}$

C $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} - \underset{\sim}{d}$ **D** $\underset{\sim}{a} + \underset{\sim}{c} = -\underset{\sim}{b} - \underset{\sim}{d}$

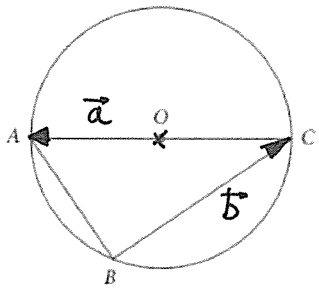


4 Consider the circle with centre O and radius $\overrightarrow{OA} = \underset{\sim}{a}$. B and C are points on the circle and $\overrightarrow{BC} = \underset{\sim}{b}$.

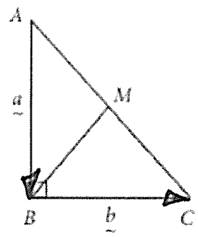
Which one of the following statements must be true?

A $\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{b}$ **B** $\underset{\sim}{a} = -\frac{1}{2}\underset{\sim}{b}$

C $\underset{\sim}{a} \bullet \underset{\sim}{b} = \underset{\sim}{b} \bullet \underset{\sim}{b}$ **D** $2\underset{\sim}{a} \bullet \underset{\sim}{b} = -\underset{\sim}{b} \bullet \underset{\sim}{b}$



5 Use vector methods to prove that the midpoint of the hypotenuse of a right-angled triangle is equidistant from all vertices.

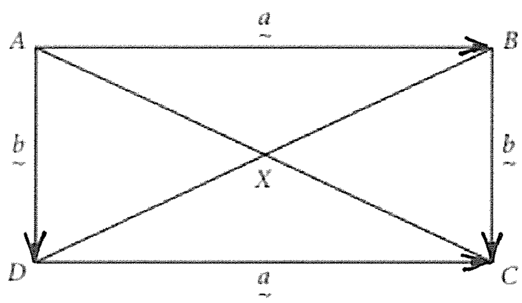


VECTORS IN GEOMETRIC PROOFS

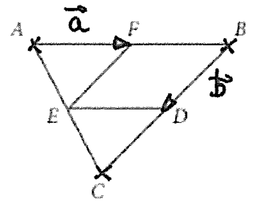
7 *ABCD* is a rectangle.

(a) Prove that the diagonals of a rectangle bisect each other.

(b) Prove that the diagonals of a rectangle are equal in length.



VECTORS IN GEOMETRIC PROOFS



8 *BDEF* is a parallelogram contained within a triangle *ABC*, as shown.

Let $\overrightarrow{AF} = \underset{\sim}{a}$, $\overrightarrow{BD} = \underset{\sim}{b}$ and F be the midpoint of $\overline{AB}$.

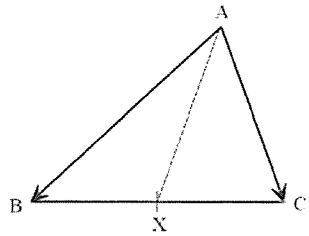
(a) Find the vector $\overrightarrow{AE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(b) Use vector methods to prove that $\overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC}$.

9 ABC is a triangle with $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{AC} = \underset{\sim}{b}$. X is the midpoint of BC as shown.

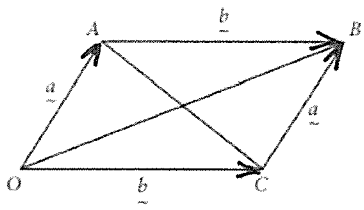
(a) Find $\overrightarrow{BX}$ and $\overrightarrow{AX}$.

(b) Find $2\left(\overrightarrow{BX} \bullet \overrightarrow{BX} + \overrightarrow{AX} \bullet \overrightarrow{AX}\right)$.



Apollonius' theorem relates to the length of a median of a triangle to the lengths of its sides. In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.

(c) Prove this, i.e. prove that $\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 = 2\left(\left|\overrightarrow{AX}\right|^2 + \left|\overrightarrow{BX}\right|^2\right)$.



10 Consider the parallelogram $OABC$ where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{b}$.

(a) Find the diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(b) Find the sum of the squares of the lengths of the sides of the parallelogram in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(c) Find the sum of the squares of the lengths of the diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(d) Hence prove that the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides.