SOLVING QUADRATIC TRIGONOMETRIC EQUATIONS

1 Solve for values between 0 and 2π inclusive:

(a) $\tan^2 x - 1 = 0$ **(b)** $\sin^2 x - \sin x = 0$ **(c)** $\cos^2\theta - 2\cos\theta + 1 = 0$

a) $\tan^2 x = 1$ so either 1) $\tan x = 1$ or 2) $\tan x = -1$

for 1) $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

for 2) $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

b) $\sin x(\sin x - 1) = 0$ so either $\sin x = 0$ (i.e $x = 0$ or π or 2π)

or $\sin x = 1$ so $x = \frac{\pi}{2}$

c) $\Rightarrow (\cos\theta - 1)^2 = 0$

So $\cos\theta = 1$ $\theta = 0$ or $\theta = 2\pi$

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3 Solve for values between 0 and 2π inclusive:

(a) $2\cos^2\theta - 3\cos\theta - 2 = 0$ **(b)** $2\cos^2\theta + \sin\theta = 1$

a) (E)

say $x = \cos\theta$ (E) $\Leftrightarrow 2x^2 - 3x - 2 = 0$

$\Delta = 9 - 4 \times (-2) \times 2 = 25 = 5^2$

So either $x = \frac{3-5}{4} = -\frac{1}{2}$ or $x = \frac{3+5}{4} = 2$

$x = 2$ is impossible as $-1 \leq \cos\theta \leq 1$

So $x = -\frac{1}{2} = \cos\theta$ $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$

b) $2\cos^2\theta + \sin\theta = 1 \Leftrightarrow 2(1 - \sin^2\theta) + \sin\theta = 1$

$\Leftrightarrow -2\sin^2\theta + \sin\theta + 1 = 0$

$\Delta = 1 + 4 \times 2 = 9 = 3^2$ $x = \sin\theta$ $-2x^2 + x + 1 = 0$

$x = \frac{-1+3}{(-4)} = \frac{2}{(-4)} = -\frac{1}{2}$ or $x = \frac{-1-3}{-4} = +1$

so $\sin\theta = -\frac{1}{2}$ i.e. $\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

or $\sin\theta = 1$ i.e. $\theta = \frac{\pi}{2}$

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6 Solve for $0 \le \theta \le 2\pi$: **(a)** $3\tan^3\theta - 3\tan^2\theta - \tan\theta + 1 = 0$ **(b)** $\cos^3\theta - 2\cos^2\theta + \cos\theta = 0$

a) $\Leftrightarrow 3\tan^2\theta(\tan\theta - 1) - (\tan\theta - 1) = 0$

$\Leftrightarrow (\tan\theta - 1)(3\tan^2\theta - 1) = 0$

So either $\tan\theta = 1$, i.e. $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

OR $3\tan^2\theta = 1$, i.e. $\tan\theta = \pm\frac{1}{\sqrt{3}} = \pm\tan\frac{\pi}{6}$

i.e. $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$ or $\frac{5\pi}{6}, \frac{11\pi}{6}$

in conclusion, there are 6 solutions: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{11\pi}{6}$

b) $\Leftrightarrow \cos\theta[\cos^2\theta - 2\cos\theta + 1] = 0$

So either $\cos\theta = 0$, i.e. $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

OR $\cos^2\theta - 2\cos\theta + 1 = 0$

$\Leftrightarrow (\cos\theta - 1)^2 = 0$

$\Leftrightarrow \cos\theta = 1$ i.e. $\theta = 0, 2\pi$

So in conclusion, 4 solutions: $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$