THE DISTRIBUTIVE LAW

The distributive law can be used with surds to expand expressions with a binomial factor:

$$a(b+c) = ab + ac$$

Example 19

Expand and simplify:

(a) $\sqrt{6}\left(\sqrt{2}+2\sqrt{3}\right)$ (b) $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{5}+\sqrt{6}\right)$ (c) $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$ (d) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$

Solution

(a) $\sqrt{6}\left(\sqrt{2}+2\sqrt{3}\right)$

$= \sqrt{6}\times\sqrt{2}+\sqrt{6}\times 2\sqrt{3}$

$= \sqrt{12}+2\sqrt{18}$

$= 2\sqrt{3}+6\sqrt{2}$

(b) $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{5}+\sqrt{6}\right)$

$= \sqrt{3}\left(\sqrt{5}+\sqrt{6}\right)+\sqrt{2}\left(\sqrt{5}+\sqrt{6}\right)$

$= \sqrt{15}+3\sqrt{2}+\sqrt{10}+2\sqrt{3}$

(c) $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$

$= \left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$

$= 5-3$

$= 2$

This is similar to $(a-b)(a+b) = a^2 - b^2$.

(d) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$

$= \sqrt{5}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)$

$= 5+2\sqrt{15}+3$

$= 8+2\sqrt{15}$

This is similar to $(a+b)^2 = a^2 + 2ab + b^2$.