PARABOLAS AND DISCRIMINANTS

2 Calculate the discriminant for each of the following equations and hence state whether the equations have two, one or no real roots.

(a) $x^2 + 6x + 2 = 0$ (b) $2x^2 + 3x + 4 = 0$ (c) $4x^2 - 12x + 9 = 0$

(d) $-3x^2 + 2x - 1 = 0$ (e) $2x^2 = 3x + 7$

a) $\Delta = b^2 - 4ac = 6^2 - 4 \times 1 \times 2 = 28$ which is positive, so 2 solutions.

b) $\Delta = 3^2 - 4 \times 2 \times 4 = 9 - 32 = -23$ which is negative, so 0 solutions.

c) $\Delta = 12^2 - 4 \times 4 \times 9 = 0$ so 1 solution.

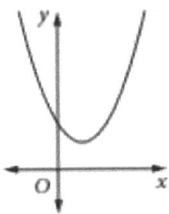d) $\Delta = 2^2 - 4 \times (-3) \times (-1) = 4 - 12 < 0$ so no solutions.

e) $\Delta = (-3)^2 - 4 \times 2 \times (-7) = 9 + .. > 0$ so 2 solutions.

3 For which curve can you say that $a > 0$ and $\Delta < 0$? → concave up → no x intercept
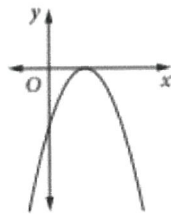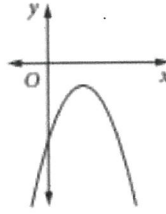
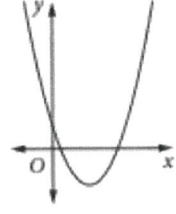A   B   C   D

4 Without sketching the graphs of each function, determine whether or not they cross the x-axis.

(a) $y = x^2 - 5x + 2$ (b) $y = -4x^2 + 2x - 1$ (c) $y = x^2 - 6x + 9$

a) $\Delta = (-5)^2 - 4 \times 1 \times 2 = 25 - 8$ which is positive

∴ it crosses the x-axis at 2 points.

b) $\Delta = 2^2 - 4 \times (-4) \times (-1) = 4 - 16 = -12 < 0$

∴ it doesn't cross the x-axis.

c) $\Delta = (-6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0$

∴ it touches the x-axis at one point.

5 Calculate the discriminant for each of the following equations. Use this information to decide on the best solution method, then solve each equation.

(a) $x^2 + 2x - 15 = 0$ (b) $x^2 - 9x - 5 = 0$ (c) $12x^2 = 25x - 12$

(m) $4x^2 = 9x - 4$ (n) $9x^2 + 24x + 16 = 0$ (o) $3x^2 + 4x = 5$

a) $\Delta = 4 - 4 \times (-15) = 64 = 8^2$ $x_1 = \dfrac{-2+8}{2} = 3$ or $x_2 = \dfrac{-2-8}{2} = -5$

b) $\Delta = (-9)^2 - 4 \times (-5) = 81 + 20 = 101$ so 2 solutions.

$x = \dfrac{9 \pm \sqrt{101}}{2}$

c) $\Longleftrightarrow$ $12x^2 - 25x + 12 = 0$ $\Delta = 25^2 - 4 \times 12 \times 12 = 49 = 7^2$

$x_1 = \dfrac{25+7}{2 \times 12} = \dfrac{32}{24} = \dfrac{4}{3}$ or $x_2 = \dfrac{25-7}{2 \times 12} = \dfrac{18}{24} = \dfrac{3}{4}$

m) $\Longleftrightarrow$ $4x^2 - 9x + 4 = 0$ $\Delta = (-9)^2 - 4 \times 4 \times 4 = 17$

$x_1 = \dfrac{9 + \sqrt{17}}{8}$ or $x_2 = \dfrac{9 - \sqrt{17}}{8}$

n) $9x^2 + 24x + 16 = 0$ $\Delta = 24^2 - 4 \times 9 \times 16 = 0$ so 1 solution.

$x = \dfrac{-24}{2 \times 9} = -\dfrac{4}{3}$

o) $\Longleftrightarrow$ $3x^2 + 4x - 5 = 0$ $\Delta = 4^2 - 4 \times 3 \times (-5) = 76$

$\sqrt{\Delta} = 2\sqrt{19}$ So 2 solutions.

$x_1 = \dfrac{-4 + 2\sqrt{19}}{2 \times 3} = \dfrac{-2 + \sqrt{19}}{3}$

or $x_2 = \dfrac{-2 - \sqrt{19}}{3}$