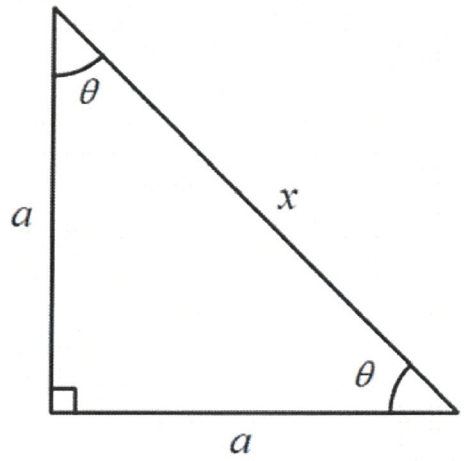
Part A: Exact values of trigonometric ratios for 45°

Consider the isosceles triangle:



a) If the length of both equal sides is a, use Pythagoras theorem to find the length of the hypotenuse x

$x^2 = a^2 + a^2 = 2a^2$

So $\boxed{x = \sqrt{2}\,a}$

b) Show that $\theta = 45°$.

$2\theta + 90 = 180$ so $2\theta = 90$ $\therefore \theta = 45°$

c) Show that $sin\ 45 = \frac{1}{\sqrt{2}}$ (which, once rationalised, is equal to $\frac{\sqrt{2}}{2}$)

$\sin 45 = \frac{a}{x} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

d) Show that $cos\ 45 = \frac{1}{\sqrt{2}}$ (which, once rationalised, is equal to $\frac{\sqrt{2}}{2}$)

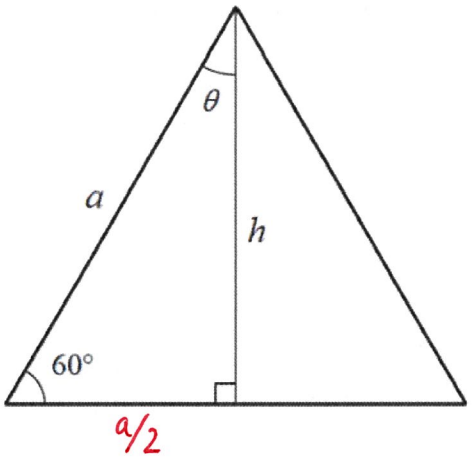
$\cos 45 = \frac{a}{x} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

e) Show that $tan\ 45 = 1$

$\tan 45 = \frac{\sin 45}{\cos 45} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

Part B: Exact values of trigonometric ratios for 30° and 60°

Consider the equilateral triangle:



a) Show that $\theta = 30°$

$90+\theta+60=180 \quad \therefore \quad \theta = 30°$

b) If the length of each side is a, use Pythagoras theorem to find the length of the height h

$h^2+\left(\frac{a}{2}\right)^2=a^2 \quad \therefore \quad h^2+\frac{a^2}{4}=a^2 \quad \therefore \quad h^2=a^2-\frac{a^2}{4}$

$h^2=\frac{3a^2}{4} \quad \therefore \quad \boxed{h=\frac{\sqrt{3}}{2}a}$

c) Using SOHCAHTOA, show that $sin\ 30 = \frac{1}{2}$ and that $cos\ 30 = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{a/2}{a} = \frac{1}{2} \qquad \cos 30 = \frac{h}{a} = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$

d) Likewise, show that $sin\ 60 = \frac{\sqrt{3}}{2}$ and that $cos\ 60 = \frac{1}{2}$

$\sin 60 = \frac{h}{a} = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{a/2}{a} = \frac{1}{2}$

e) Using results from c) and d) above, find the exact values of $tan\ 30$ and of $tan\ 60$

$\tan 30 = \frac{\sin 30}{\cos 30} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan 60 = \frac{\sin 60}{\cos 60} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$