ARRANGEMENT OF n OBJECTS WHEN SOME ARE IDENTICAL

In how many ways can the six letters of the word MAMMAL be arranged in a line?

Solution

There are six letters, of which three are 'M' and two are 'A'.

$$\therefore \text{ Number of arrangements} = \frac{6!}{3! \times 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2}{3 \times 2 \times 2} = 60$$

One of the 60 arrangements is MMMAAL. If the letters 'M' were different, e.g. M_1, M_2, M_3, then this arrangement could be six different arrangements (3!):

$M_1M_2M_3AAL$ $M_1M_3M_2AAL$ $M_2M_1M_3AAL$ $M_2M_3M_1AAL$ $M_3M_1M_2AAL$ $M_3M_2M_1AAL$

Without the subscripts, all these arrangements are the same. This is why 6! is divided by 3! in the calculation of identical object arrangements.

Similarly, the two letters 'A' have 2! arrangements the same, so the result is also divided by 2!.

The number of ways of arranging n objects in a row when p of the objects are identical, and q of the objects are identical (but different to the others), and so on, is not $n!$ but instead:

$$\frac{n!}{p!\,q!}$$