THE ANTI-DERIVATIVE - CHAPTER REVIEW

1 Find the primitive of the following:

(a) $x+9$ **(b)** $3x^2-2x+4$ **(c)** x^4+x^3-2 **(d)** $(x-2)(x+3)$ **(e)** $(x+2)^2$ **(f)** 7

a) $\int (x+9)\,dx = \frac{x^2}{2} + 9x + C$

b) $\int (3x^2-2x+4)\,dx = x^3 - x^2 + 4x + C$

c) $\int (x^4+x^3-2)\,dx = \frac{x^5}{5} + \frac{x^4}{4} - 2x + C$

d) $\int (x-2)(x+3)\,dx = \int (x^2+x-6)\,dx = \frac{x^3}{3} + \frac{x^2}{2} - 6x + C$

e) $\int (x+2)^2\,dx = \int (x^2+4x+4)\,dx = \frac{x^3}{3} + 2x^2 + 4x + C$

f) $\int 7\,dx = 7x + C$

2 Express y in terms of x, given the following:

(a) $\frac{dy}{dx} = 5x+4$ **(b)** $\frac{dy}{dx} = 5-4x+3x^2+x^3$ **(c)** $\frac{dy}{dx} = 2x+\sqrt{x}+3$

$y = \int (5x+4)\,dx = 5\frac{x^2}{2} + 4x + C$

b) $y = \int (5-4x+3x^2+x^3)\,dx = 5x - 4\frac{x^2}{2} + x^3 + \frac{x^4}{4} + C$

c) $y = \int (2x+\sqrt{x}+3)\,dx = x^2 + \frac{2x^{3/2}}{3} + 3x + C$

3 Find $f(x)$ in terms of x, given the following:

(a) $f'(x) = x^2 + x^3 + 1, f(0) = 2$ **(b)** $f'(x) = 3 - x + 6x^3, f(1) = 3$ **(c)** $f'(x) = 1 - \frac{1}{x^2}, f(2) = \frac{1}{2}$

a) $f(x) = \int x^2 + x^3 + 1 = \frac{x^3}{3} + \frac{x^4}{4} + x + C$

$f(0) = 2$ so $2 = \frac{0^3}{3} + \frac{0^4}{4} + 0 + C$ so $C = 2$

b) $f(x) = \int (3 - x + 6x^3)\,dx = 3x - \frac{x^2}{2} + \frac{6x^4}{4} + C$

$f(1) = 3$ so $3 = 3 - \frac{1}{2} + \frac{6}{4} + C$ so $C = -1$

c) $f(x) = \int \left(1 - \frac{1}{x^2}\right) dx = x - \frac{x^{-1}}{(-1)} + C = x + \frac{1}{x} + C$

$f(2) = \frac{1}{2}$ so $\frac{1}{2} = 2 + \frac{1}{2} + C$ so $C = -2$

4 During a storm, water flows into a 5000-litre tank at the rate of $\frac{dV}{dt}$ litres per minute, where $\frac{dV}{dt} = 140 + 13t - t^2$ and t is the time in minutes since the storm began.

(a) Find the volume of water that has flowed into the tank since the start of the storm as a function of t.

(b) How much water has flowed into the tank after 12 minutes?

a) $V(t) = \int (140 + 13t - t^2)\,dt = 140t + \frac{13t^2}{2} - \frac{t^3}{3} + C$

At $t = 0$ $V = 0$ so $C = 0$

$V(t) = 140t + \frac{13t^2}{2} - \frac{t^3}{3}$

b) $V(12) = 140 \times 12 + 13 \times \frac{12^2}{2} - \frac{12^3}{3}$

$V(12) = 2040$

5 **(a)** Show that $\frac{d}{dx}\left(xe^x\right)=e^x+xe^x$. **(b)** Hence find $\int xe^x\,dx$.

a) $\frac{d}{dx}\left(xe^x\right)=\frac{d}{dx}\left(u(x)\,v(x)\right)$ $u(x)=x$ $u'(x)=1$

Product rule $v(x)=e^x$ $v'(x)=e^x$

$\frac{d}{dx}\left(xe^x\right)=1\times e^x+x\,e^x=e^x+xe^x$

b) $\int xe^x\,dx=\int\left[\frac{d}{dx}\left(xe^x\right)-e^x\right]dx=\int\frac{d}{dx}\left(xe^x\right)dx-\int e^x\,dx$

$=xe^x-e^x+C$

$\boxed{4}$ **6** Find: **(a)** $\int 3\sin\frac{x}{2}\,dx$ **(b)** $\int\left(x+\sec^2 2x\right)dx$ **(c)** $\int\frac{\cos t}{\sin t}\,dt$

a) $\int 3\sin\frac{x}{2}\,dx=-3\cos\left(\frac{x}{\cancel{2}}\right)\times 2+C=-6\cos\left(\frac{x}{2}\right)+C$

b) $\int\left[x+\sec^2(2x)\right]dx=\frac{x^2}{2}+\frac{\tan(2x)}{2}+C$

c) $\int\frac{\cos t}{\sin t}\,dt=\int\cos t\times\frac{1}{\sin t}\,dt=\int\frac{d}{dt}\left(\sin t\right)\times\frac{1}{\sin t}\,dt$

$=\ln|\sin(t)|+C$

so $\int\cot(t)\,dt=\ln|\sin t|+C$

7 Find: (a) $\int \frac{5}{x}\,dx$ (b) $\int \frac{3}{x+4}\,dx$ (c) $\int \frac{4x}{x^2+1}\,dx$ (d) $\int \frac{e^x}{e^x+2}\,dx$

a) $\int \frac{5}{x}\,dx = 5\ln|x| + C$

b) $\int \frac{3}{x+4}\,dx = 3\ln|x+4| + C$

c) $\int \frac{4x}{x^2+1}\,dx = 2\int \frac{2x}{x^2+1}\,dx = 2\ln|x^2+1| + C$

d) $\int \frac{e^x}{e^x+2}\,dx = \ln|e^x+2| + C$