COMPLETING THE SQUARE FOR NON-MONIC EQUATIONS

Complete the square to solve the following quadratic equations, giving your answers in surd form.

1 $2x^2 - x - 5 = 0$ **2** $2x^2 + 6x - 5 = 0$ **3** $2x^2 + x - 2 = 0$

(1) $\Leftrightarrow x^2 - \frac{x}{2} - \frac{5}{2} = 0 \Leftrightarrow \left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{5}{2} = 0$

$\Leftrightarrow \left(x - \frac{1}{4}\right)^2 = \frac{41}{16}$

$x - \frac{1}{4} = \pm\sqrt{\frac{41}{16}} = \pm\frac{\sqrt{41}}{4}$

$\therefore \quad x = \frac{\sqrt{41}}{4} - \frac{1}{4}$ or $x = -\frac{\sqrt{41}}{4} - \frac{1}{4}$

(2) $\Leftrightarrow x^2 + 3x - \frac{5}{2} = 0 \Leftrightarrow \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{5}{2} = 0$

$\Leftrightarrow \left(x + \frac{3}{2}\right)^2 = \frac{19}{4}$

$\therefore \quad x + \frac{3}{2} = \pm\sqrt{\frac{19}{4}} = \pm\frac{\sqrt{19}}{2}$

$x = \frac{\sqrt{19}}{2} - \frac{3}{2}$ or $x = \frac{-\sqrt{19}}{2} - \frac{3}{2}$

(3) $\Leftrightarrow x^2 + \frac{x}{2} - 1 = 0$

$\Leftrightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 1 = 0 \Leftrightarrow \left(x + \frac{1}{4}\right)^2 = \frac{17}{4}$

$\therefore \quad x + \frac{1}{4} = \pm\sqrt{\frac{17}{4}} = \pm\frac{\sqrt{17}}{2}$

$\therefore \quad x = \frac{\sqrt{17}}{2} - \frac{1}{4}$ or $x = \frac{-\sqrt{17}}{2} - \frac{1}{4}$

COMPLETING THE SQUARE FOR NON-MONIC EQUATIONS

10 $4x^2+4x-5=0$ **11** $2x^2-5x=9$ **12** $3x^2-2x-2=0$

(10) $x^2+x-\frac{5}{4}=0 \iff \left(x+\frac{1}{2}\right)^2-\frac{1}{4}-\frac{5}{4}=0$

$\iff \left(x+\frac{1}{2}\right)^2-\frac{6}{4}=0 \iff \left(x+\frac{1}{2}\right)^2=\frac{3}{2}$

so $x+\frac{1}{2}=\pm\sqrt{\frac{3}{2}}$

$x=\sqrt{\frac{3}{2}}-\frac{1}{2}$ or $x=-\sqrt{\frac{3}{2}}-\frac{1}{2}$

(11) $x^2-\frac{5}{2}x=\frac{9}{2} \iff \left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2=\frac{9}{2}$

$\iff \left(x-\frac{5}{4}\right)^2=\frac{97}{16}$

$\therefore\ x=\pm\sqrt{\frac{97}{16}}+\frac{5}{4}=\pm\frac{\sqrt{97}}{4}+\frac{5}{4}$

$x=\frac{\sqrt{97}}{4}+\frac{5}{4}$ or $x=-\frac{\sqrt{97}}{4}+\frac{5}{4}$

(12) $x^2-\frac{2}{3}x-\frac{2}{3}=0 \iff \left(x-\frac{1}{3}\right)^2-\frac{1}{9}-\frac{2}{3}=0$

$\iff \left(x-\frac{1}{3}\right)^2=\frac{7}{9}$

$x-\frac{1}{3}=\pm\sqrt{\frac{7}{9}}=\pm\frac{\sqrt{7}}{3}$

$\therefore\ x=\frac{\sqrt{7}}{3}+\frac{1}{3}$ or $x=-\frac{\sqrt{7}}{3}+\frac{1}{3}$