FURTHER TRIGONOMETRY - CHAPTER REVIEW

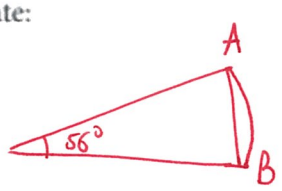
1 The minute hand of a clock is 1.2 metres long. How far does the tip move in 40 minutes?

in 40 minutes, it rotates $\frac{40}{60} = \frac{2}{3}$ of a full revolution, so $\frac{2}{3} \times 2\pi = \frac{4\pi}{3}$ radians

$$l = \frac{4\pi}{3} \times 1.2 = 5.03 \text{ m}$$

2 An arc AB of length 6 cm subtends an angle of 56° at the centre of a circle. Calculate:

(a) the length of the radius **(b)** the length of the chord AB.



a) $l = r\theta$ so $r = \frac{l}{\theta} = \frac{6}{\pi 56/180} = \frac{180 \times 6}{56\pi} = \frac{135}{7\pi} \simeq 6.1 \text{ cm}$

b) $AB^2 = 2\left(\frac{135}{7\pi}\right)^2 - 2\left(\frac{135}{7\pi}\right)\cos 56$ (cosine rule)

$AB \simeq 5.73 \text{ cm}$

3 Find the exact value of $\sec^2 \frac{\pi}{4} + \operatorname{cosec}^2 \frac{\pi}{4}$.

$$\sec^2 \pi/4 + \operatorname{cosec}^2 \pi/4 = \frac{1}{\cos^2 \pi/4} + \frac{1}{\sin^2 \pi/4}$$

$$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= 2 \times \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = 2 \times \frac{1}{\frac{2}{4}} = 2 \times \frac{1}{\frac{1}{2}} = 4$$

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4 Solve, for $-\pi \leq x \leq \pi$, $6\cos^2 x - 5\cos x + 1 = 0$.

we do a change of variable $X = \cos x$, so the equation becomes:

$6X^2 - 5X + 1 = 0$ $\quad \Delta = (-5)^2 - 4 \times 6 = 25 - 24 = 1 > 0$ 2 solutions

$X_1 = \frac{5+1}{2\times 6} = \frac{1}{2}$ or $X_2 = \frac{5-1}{2\times 6} = \frac{4}{12} = \frac{1}{3}$

if $\cos x = \frac{1}{2}$ then $x = -\frac{\pi}{3}$ or $x = \frac{\pi}{3}$

if $\cos x = \frac{1}{3}$ Then $x = 70.53°$ or $x = -70.53°$

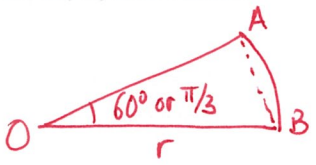
$x = 1.23$ rad or $x = -1.23$ rad.

5 An arc AB of a sector of a circle is $\frac{\pi}{4}$ metres long and subtends an angle of 60° at the centre, O, of the circle.

Calculate: **(a)** the length of the radius

(b) the area of the sector AOB (correct to 1 decimal place)

(c) the length of the chord AB (correct to 1 decimal place).



a) $\frac{\pi}{4} = r \times \frac{\pi}{3}$ so $r = 3/4$ m

b) Area $= \frac{1}{2} \times \left(\frac{3}{4}\right)^2 \times \frac{\pi}{3} = \frac{9\pi}{96} = \frac{3\pi}{32}$

c) $AB^2 = 2 \times \left(\frac{3}{4}\right)^2 - 2 \times \left(\frac{3}{4}\right)^2 \cos(\pi/3) = \frac{9}{8} - \frac{9}{8} \times \frac{1}{2} = \frac{9}{16}$

so $AB = \sqrt{\frac{9}{16}} = \frac{3}{4}$

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6 Find all values of θ between 0 and 2π for which:

(a) $\sin\theta = -0.5$ **(b)** $\cos\theta = 0$ **(c)** $\tan\theta = -1$

(d) $\sec\theta = \frac{2}{\sqrt{3}}$ **(e)** $\cot\theta = \sqrt{3}$ **(f)** $\operatorname{cosec}\theta = \sqrt{2}$

a) $\sin\theta = -0.5$ $\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

b) $\cos\theta = 0$ so $\theta = \pi/2$ or $\theta = \frac{3\pi}{2}$

c) $\tan\theta = -1$ so $\theta = \frac{3\pi}{4}$ or $\theta = \frac{7\pi}{4}$

d) $\sec\theta = \frac{2}{\sqrt{3}} \Leftrightarrow \cos\theta = \frac{\sqrt{3}}{2}$ so $\theta = \frac{\pi}{6}$ or $\theta = \frac{11\pi}{6}$

e) $\cot\theta = \sqrt{3}$ so $\tan\theta = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \frac{-1/2}{-\sqrt{3}/2}$

so $\theta = \frac{\pi}{6}$ or $\theta = \frac{7\pi}{6}$

f) $\operatorname{cosec}\theta = \sqrt{2}$ so $\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

so $\theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$

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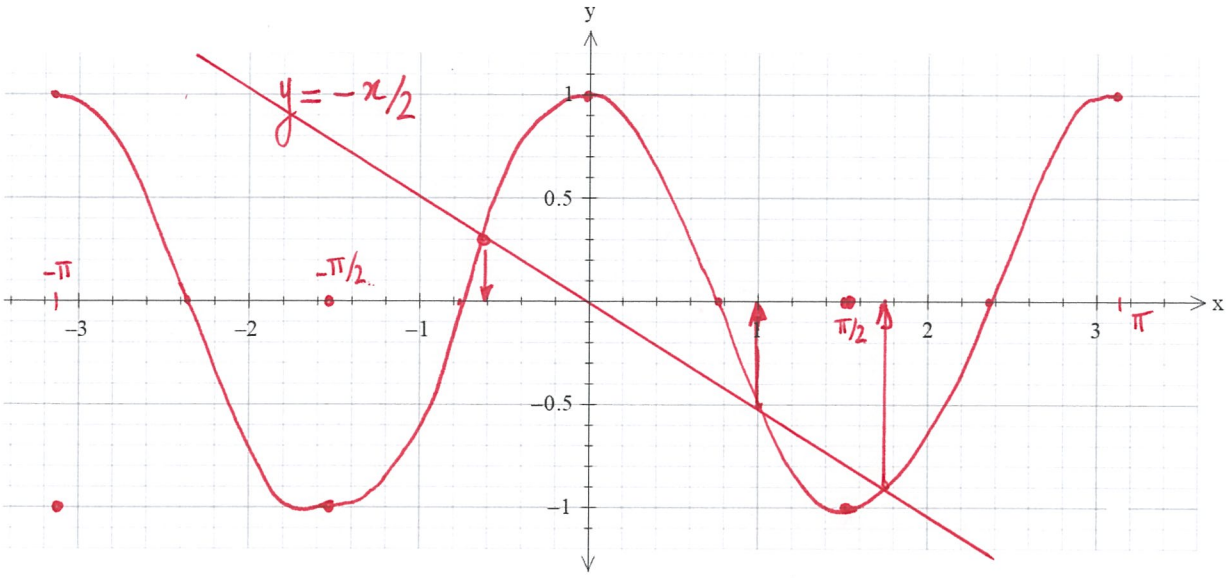
7 Draw the graph of $y = \cos 2x$ for $-\pi \leq x \leq \pi$. On the same set of axes, draw the graph of $y = -\frac{x}{2}$. Use your graphs to solve the equation $\cos 2x = -\frac{x}{2}$.



$x \approx -0.6$ or $x \approx 1$ or $x \approx 1.8$

9 Express in radians, in terms of π: (a) 45° (b) 240° (c) 160° (d) −210°

a) $45° = \frac{\pi}{4}$ radians

b) $240° = \frac{4\pi}{3}$ radians

c) $160° = 160 \times \frac{\pi}{180} = \frac{8\pi}{9}$

d) $-210° = -210 \times \frac{\pi}{180} = -\frac{7\pi}{6}$

10 Express in radians, correct to 4 decimal places: (a) 65° (b) 281° (c) −100° (d) −326°

a) $65 \times \frac{\pi}{180} = 1.1345 \text{rad}$

b) $281 \times \frac{\pi}{180} = 4.9044 \text{rad}$

c) $-100 \times \frac{\pi}{180} = -1.7453 \text{rad}$

d) $-326 \times \frac{\pi}{180} = -5.6898 \text{ rad}$

11 Express in degrees the angles whose radian measures are: (a) $\frac{4\pi}{5}$ (b) $\frac{7\pi}{6}$ (c) $\frac{23\pi}{12}$ (d) $-\frac{3\pi}{2}$

a) $\frac{4\pi}{5} \times \frac{180}{\pi} = 144°$

b) $\frac{7\pi}{6} \times \frac{180}{\pi} = 210°$

c) $\frac{23\pi}{12} \times \frac{180}{\pi} = 345°$

d) $-\frac{3\pi}{2} \times \frac{180}{\pi} = -270°$

12 Express in degrees and minutes, to the nearest minute, the angles whose radian measures are:

(a) 2.6 **(b)** −1.4 **(c)** 0.341 **(d)** −3

a) $2.6 \times \frac{180}{\pi} = 148°58'$

b) $-1.4 \times \frac{180}{\pi} = -80°13'$

c) $0.341 \times \frac{180}{\pi} = 19°32'$

d) $-3 \times \frac{180}{\pi} = -171°53'$

13 Simplify: **(a)** $\sin(\pi + x)$ **(b)** $\cos(2\pi - x)$ **(c)** $\tan(\pi - x)$

a) $\sin(\pi + x) = -\sin x$

b) $\cos(2\pi - x) = \cos(-x) = \cos x$

c) $\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)} = \frac{\sin x}{-\cos x} = -\tan x$

14 Write the exact value of: **(a)** $\cos\pi$ **(b)** $\tan\frac{7\pi}{6}$ **(c)** $\sin\frac{3\pi}{4}$ **(d)** $\cos\frac{5\pi}{3}$

a) $\cos\pi = -1$

b) $\tan\frac{7\pi}{6} = \frac{\sin(7\pi/6)}{\cos(7\pi/6)} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

c) $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

d) $\cos\frac{5\pi}{3} = \frac{1}{2}$

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15 Solve for $0 < x < 2\pi$: (a) $\sin x = -\frac{1}{2}$ (b) $\sin x = \sqrt{3}\cos x$ (c) $\sqrt{2}\cos x + 1 = 0$

a) $\sin x = -\frac{1}{2} \iff x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

b) $\sin x = \sqrt{3}\cos x \iff \tan x = \sqrt{3} = \frac{\sqrt{3}/2}{1/2} = \frac{-\sqrt{3}/2}{-1/2}$

so $x = \frac{\pi}{3}$ or $x = \frac{4\pi}{3}$

c) $\sqrt{2}\cos x + 1 = 0 \iff \cos x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$x = \frac{3\pi}{4}$ or $x = \frac{5\pi}{4}$

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16 Simplify: **(a)** $\sin\left(\frac{\pi}{2}-x\right)$ **(b)** $\cos\left(\frac{3\pi}{2}+x\right)$ **(c)** $\tan\left(\frac{\pi}{2}+x\right)$

a) $\sin\left(\frac{\pi}{2}-x\right) = \cos x$

b) $\cos\left(\frac{3\pi}{2}+x\right) = \cos\left(2\pi-\frac{\pi}{2}+x\right)$

$\_\_\_\_\_ = \cos\left(-\frac{\pi}{2}+x\right) = -\sin x$

c) $\tan\left(\frac{\pi}{2}+x\right) = \frac{\sin\left(\pi/2+x\right)}{\cos\left(\pi/2+x\right)} = \frac{\cos x}{-\sin x} = -\cot an\, x$