OTHER EXAMPLES OF MOTION

Displacement, velocity, acceleration—important connections

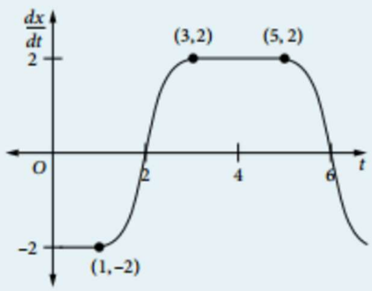
Given the displacement function, you can find the velocity and acceleration functions by differentiating with respect to time.

Given the velocity function, you can find the displacement function by integrating with respect to time, and you can find the acceleration function by differentiating with respect to time.

If the information is given as a graph, then you must remember that the definite integral can be seen as the area under the graph, while the derivative can be seen as the gradient of the curve. If you have the graph of the velocity $\frac{dx}{dt}$ against time, then the value of the definite integral gives the displacement and the slope of the curve gives the acceleration.

Example 16

The graph shows the velocity $\frac{dx}{dt}$ of a particle as a function of time. Initially the particle is at the origin.



(a) At what times is the velocity zero?

(b) At what time is the displacement x from the origin a maximum?

(c) What is the displacement when $t = 4$? What does this tell you?

(d) When does the particle have zero acceleration?

(e) At what time is the acceleration the greatest?

(f) Use the trapezoidal rule to estimate the displacement when $t = 6$.

Solution

(a) $\frac{dx}{dt} = 0$: graph shows this at $t = 2, 6$.

Hence the velocity is zero (particle at rest) at 2 seconds and again at 6 seconds.

(b) Maximum and minimum displacement occur when $\frac{dx}{dt} = 0$.

From the graph, the value of $x = \int_0^2 \frac{dx}{dt}\,dt < 0$ so the displacement at $t = 2$ is a minimum.

The value of $x = \int_0^6 \frac{dx}{dt}\,dt > 0$ so the displacement at $t = 6$ is a maximum.

(c) From the symmetry of the graph, $x = \int_0^4 \frac{dx}{dt}\,dt = 0$, so the displacement is zero and the particle is again at the origin at $t = 4$.

(d) The particle has zero acceleration when the velocity is constant, i.e. when the graph is horizontal. Acceleration is zero for $0 \le t \le 1$, $3 \le t \le 5$.

(e) The velocity graph is steepest at $t = 2$ and $t = 6$. At $t = 2$, the slope > 0; at $t = 6$, the slope < 0. Greatest acceleration is at $t = 2$.

(f) As the particle is back at the origin when $t = 4$, the displacement when $t = 6$ is given by $x = \int_4^6 \frac{dx}{dt}\,dt$.

From the graph, you have the points (4,2), (5,2), (6,0): $x = \int_4^6 \frac{dx}{dt}\,dt \approx \frac{1}{2}(2 + 2 \times 2 + 0) = 3$.

The particle is about 3 units on the positive side of the origin.