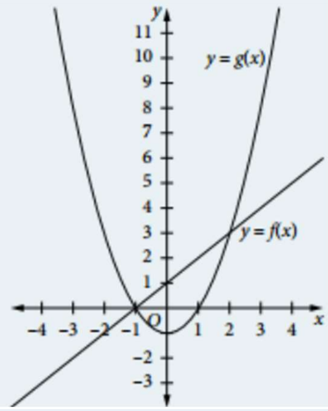
GRAPHING POLYNOMIALS BY MULTIPLYING ORDINATES

Given the graphs of two polynomial functions, $y = f(x)$ and $y = g(x)$, the graph of a new function $y = f(x)g(x)$ can be obtained by a process of multiplying the ordinates for each x value. This process is demonstrated in the following examples.

Example 13

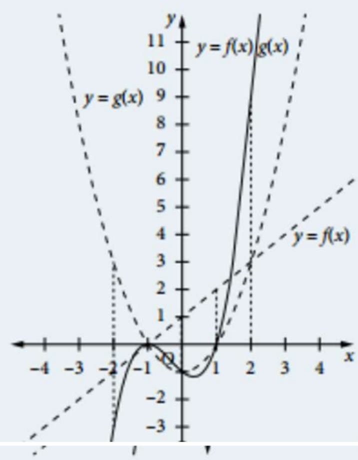
The graphs of $y = f(x)$ and $y = g(x)$ are shown. By drawing vertical lines and multiplying ordinates, draw the graph of $y = f(x)g(x)$. Comment on the new curve.



Solution

On the diagram, vertical lines are drawn through important points, such as turning points, points where a curve cuts the axes, points where the curves intersect, and points where the function value is 1 or −1.

On each vertical line, the intercepts of the two curves are multiplied to find the position of a new point, which is marked on the line.



- $x = -2$: Ordinates are 3 and −1, so $3 \times (-1) = -3$. The point on $f(x)g(x)$ is $(-2, -3)$.
- $x = -1$: Ordinates are 0 and 0, so the point on $f(x)g(x)$ is $(-1, 0)$.
- $x = 0$: Ordinates are −1 and 1, so $(-1) \times 1 = -1$. The point on $f(x)g(x)$ is $(0, -1)$.

This makes $(-1, 0)$ a local maximum turning point.

- $x = 1$: Ordinates are 0 and 2, so the point on $f(x)g(x)$ is $(1, 0)$.
- $x = 2$: Ordinates are 3 and 3, so $3 \times 3 = 9$. The point on $f(x)g(x)$ is $(2, 9)$.
- $x = 0.5$: Ordinates are approximately −0.8 and 1.5 $(-0.8) \times 1.5 = -1.2$.
  A point on $f(x)g(x)$ is $(-0.5, -1.2)$. The minimum turning point of $y = f(x)\,g(x)$ will be near this point.

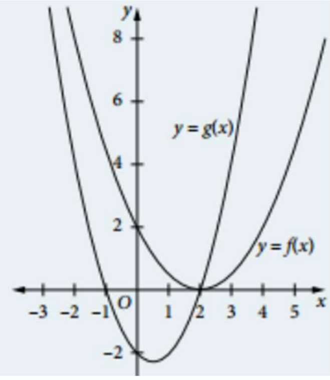
The new points are joined to obtain $y = f(x)g(x)$.

The solid curve is the graph of $y = f(x)g(x)$.

As $x \to \infty$, $f(x)g(x) \to \infty$. As $x \to -\infty$, $f(x)g(x) \to -\infty$.

Example 14

The graphs of $y = f(x)$ and $y = g(x)$ are shown. By drawing vertical lines and multiplying ordinates, draw the graph of $y = f(x)g(x)$. Comment on the new curve.



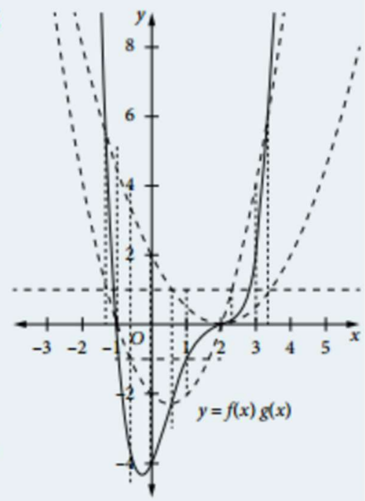
Solution

On the diagram, draw vertical lines through important points, such as turning points, points where a curve cuts the axes, where the curves intersect, and points where the function value is 1 or −1.

Multiply the intercepts on these vertical lines of the two curves and mark a new point.

Where the horizontal line $y = 1$ intersects a curve it shows the x value for which the product of the two functions is the same as the other function value.

Where the horizontal line $y = -1$ intersects a curve it shows the x value for which the product of the two functions has the opposite sign to the other function value.



- $x = -1$: Ordinates are 0 and 4.5, so the point on $f(x)g(x)$ is $(-1, 0)$.
- $x = 0$: Ordinates are −2 and 2, so $(-2) \times 2 = -4$. The point on $f(x)g(x)$ is $(0, -4)$.
- $x = 1$: Ordinates are −2 and 0.5, so $(-2) \times 0.5 = -1.5$. The point on $f(x)g(x)$ is $(1, -1.5)$.
- $x = 2$: Ordinates are 0 and 0, so the point on $f(x)\,g(x)$ is $(2, 0)$.
- $x = 3$: Ordinates are 4 and 0.5, so $4 \times 0.5 = 2$. The point on $f(x)g(x)$ is $(3, 2)$.

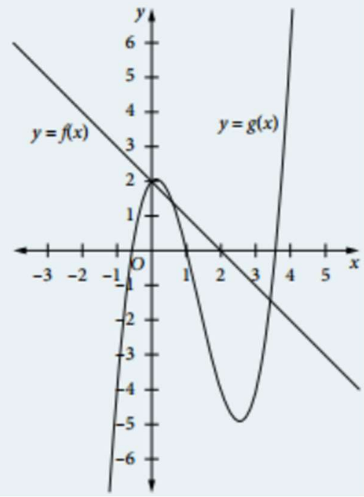
Join the points to obtain $y = f(x)g(x)$.

The solid curve is the graph of $y = f(x)g(x)$.

The new curve is a quartic, the product of two quadratic functions. It is positive where the original functions both have the same sign, zero where at least one of them is zero, and negative where only one of the functions is negative. At $(2, 0)$, where both $f(x)$ and $g(x)$ are zero, $f(x)\,g(x)$ has a horizontal point of inflection.

Example 15

The graphs of $y = f(x)$ and $y = g(x)$ are shown. By drawing vertical lines and multiplying ordinates, draw the graph of $y = f(x)\,g(x)$. Comment on the new curve.



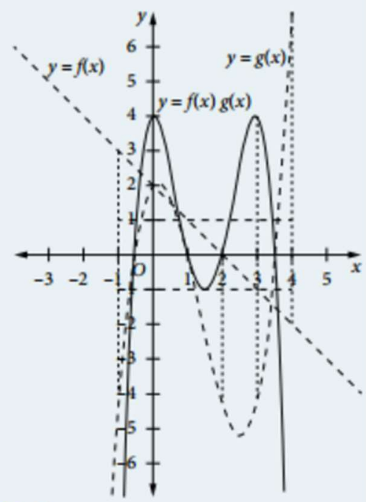
Solution

On the diagram, draw vertical lines through important points, such as turning points, points where a curve cuts the axes, where the curves intersect, and points where the function value is 1 or −1.

Multiply the intercepts on these vertical lines of the two curves and mark a new point.

Where the horizontal line $y = 1$ intersects a curve it shows the x value for which the product of the two functions is the same as the other function value.

Where the horizontal line $y = -1$ intersects a curve it shows the x value for which the product of the two functions has the opposite sign to the other function value.



- $x = -1$: Ordinates are 3 and −4, so the point on $f(x)g(x)$ is $(-1, -12)$.
- $x = 0$: Ordinates are 2 and 2, so $2 \times 2 = 4$. The point on $f(x)g(x)$ is $(0, 4)$.
- $x = 1$: $g(1) = 0$, so $f(x)g(x) = 0$ and cuts the x-axis at $(1, 0)$.
- $x = 2$: $f(2) = 0$, so $f(x)g(x) = 0$ and cuts the x-axis at $(2, 0)$.
- $x = 3$: Ordinates are (-1) and (-4), so $(-1) \times (-4) = 4$. The point on $f(x)g(x)$ is $(3, 4)$.
- $x = 4$: Ordinates are (-2) and 6, so $(-2) \times 6 = -12$. The point on $f(x)g(x)$ is $(4, -12)$.

$g(x)$ cuts the x-axis again at 0.6 and 3.6, so $f(x)g(x) = 0$ and cuts the x-axis at $(-0.6, 0)$ and $(3.6, 0)$.

Drawing $x = 1.5$ will help locate the turning point which is at approximately $(1.5, -1)$.

Join the points to obtain $y = f(x)g(x)$.

The solid curve is the graph of $y = f(x)g(x)$.

The new curve is a quartic, the product of a linear and a cubic function. It is positive where the original functions both have the same sign, zero where one of them is zero, and negative where only one of the functions is negative. As $x \to \pm\infty$, $f(x)g(x) \to -\infty$. It has a greatest value of 4, which occurs at $x = 0$ and $x = 3$.