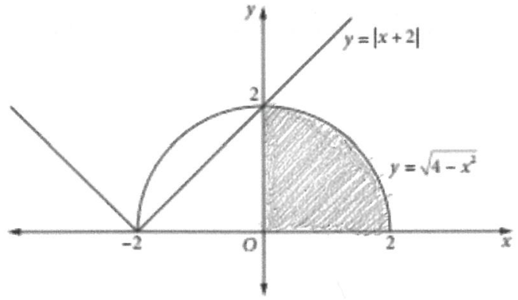
FURTHER WORK WITH FUNCTIONS - CHAPTER REVIEW

1 The diagram shows the graphs of $y=|x+2|$ and $y=\sqrt{4-x^2}$.



The solution of $\sqrt{4-x^2} \le |x+2|$ is:

$\boxed{\text{A}}$ $0 \le x \le 2$

B $-2 \le x \le 0$

C $x = -2, 0 \le x \le 2$

D $x \ge 0$

2 Solve the following inequalities.

(a) $\frac{2}{1-x} > 1$ (b) $\frac{1}{x+3} \le \frac{2}{x}$

a) $\Leftrightarrow \frac{2(1-x)^2}{(1-x)} > (1-x)^2$

$\Leftrightarrow 2(1-x) > 1-2x+x^2$

$\Leftrightarrow 2-2x > 1-2x+x^2$

$\Leftrightarrow 2 > 1+x^2$

$\Leftrightarrow x^2 < 1$

so the interval solution is $-1 < x < 1$

b) $\Leftrightarrow \frac{(x+3)^2x^2}{x+3} \le \frac{2(x+3)^2x^2}{x}$

$\Leftrightarrow (x+3)x^2 \le 2x(x+3)^2$

$\Leftrightarrow x^3+3x^2 \le 2x(x^2+6x+9)$

$\Leftrightarrow x^3+3x^2 \le 2x^3+12x^2+18x$

$\Leftrightarrow 2x^3+12x^2+18x-x^3-3x^2 \ge 0$

$\Leftrightarrow x^3+9x^2+18x \ge 0$

$\Leftrightarrow x(x^2+9x+18) \ge 0$

$\Leftrightarrow x(x+3)(x+6) \ge 0$

Case 1 if $x>0$ then we must have $(x+3)(x+6) \ge 0$ which occurs when $x \le -6$ or when $x > -3$

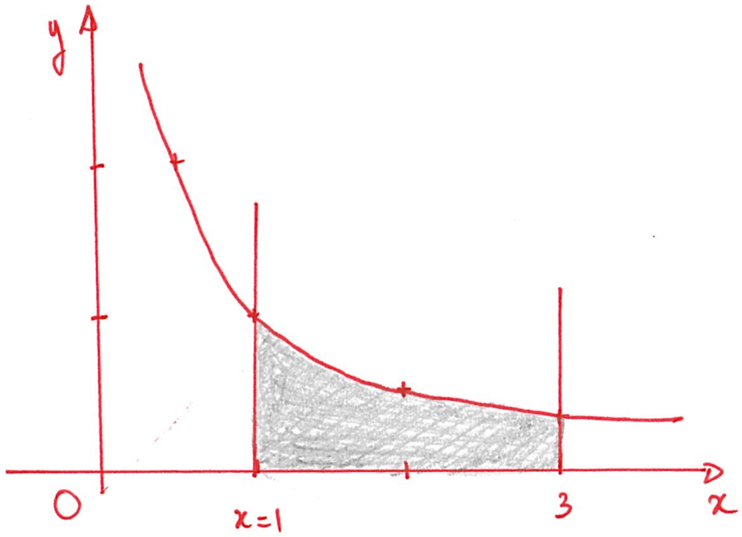
So as x must be positive, the only possible interval solution is $x>0$

Case 2 if $x<0$ Then we must have $(x+3)(x+6) \le 0$ which occurs when $-6 \le x \le -3$ (which satisfy the condition $x<0$)

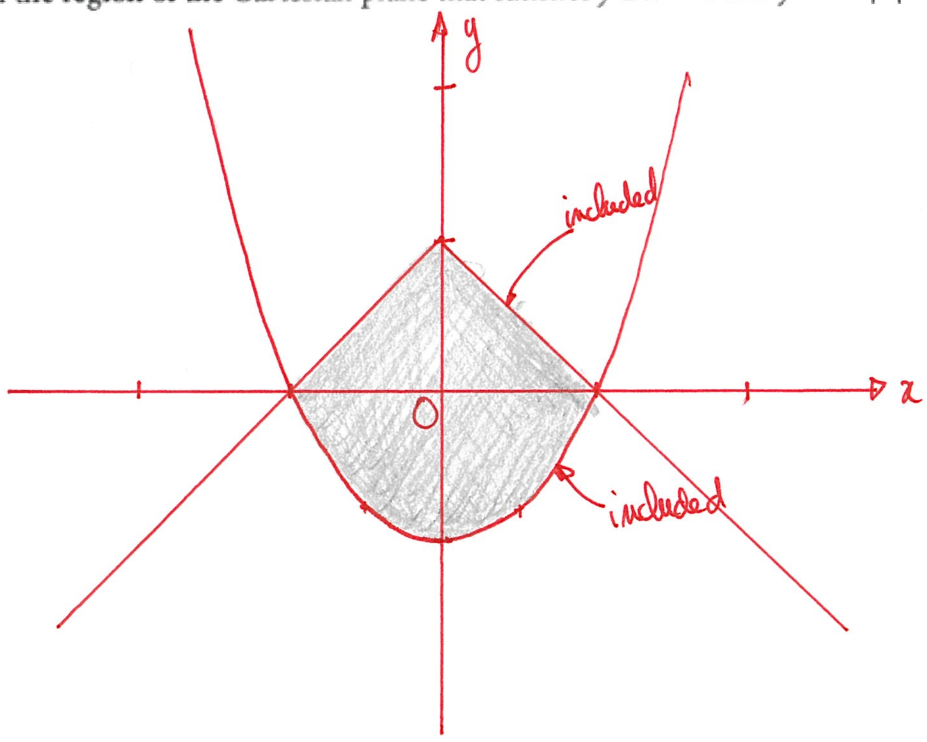
So in that case, the interval solution is $-6 \le x \le -3$

* So 2 intervals solutions : $x>0$ or $-6 \le x < 3$

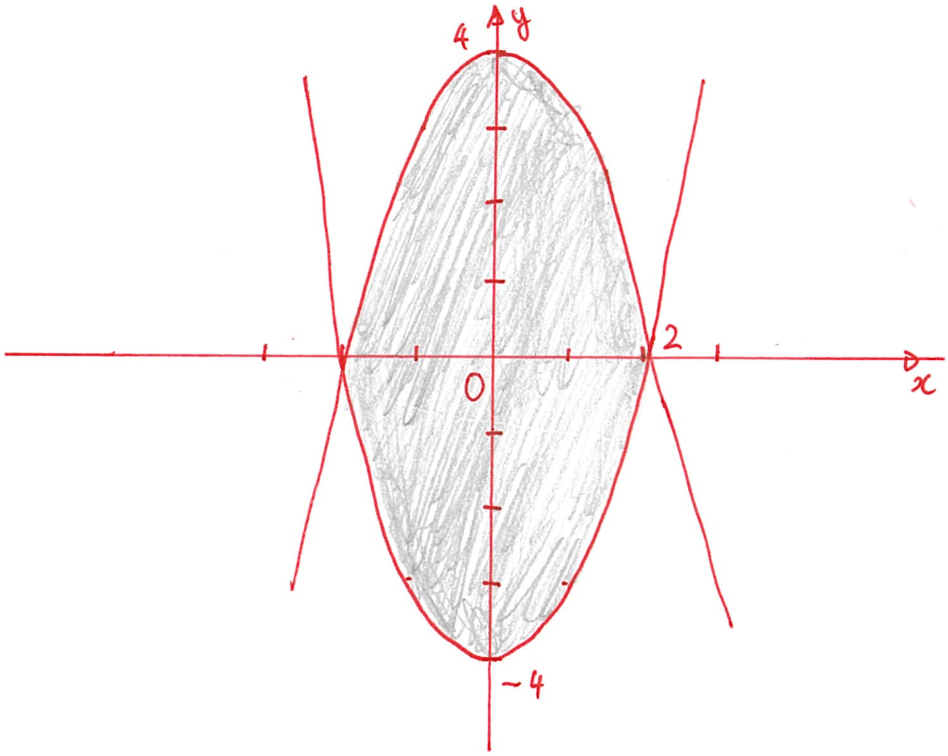
3 Sketch the region of the Cartesian plane bounded by curves $y = \frac{1}{x}$, $x = 1$, $x = 3$ and the x-axis.



4 Sketch the region of the Cartesian plane that satisfies $y \geq x^2 - 1$ and $y \leq 1 - |x|$.



5 Sketch the region of the Cartesian plane bounded by the curves $y \geq x^2 - 4$ and $y \leq 4 - x^2$.



6 Show that the straight lines $2x + y = 20$ and $x + y = 14$ intersect at (6, 8). Hence sketch the region of the Cartesian plane for which $y \geq 20 - 2x$, $y \leq 14 - x$ and $y \geq 0$ are all true.

$2x + y = 20 \iff y = -2x + 20$

$x + y = 14 \iff y = -x + 14$

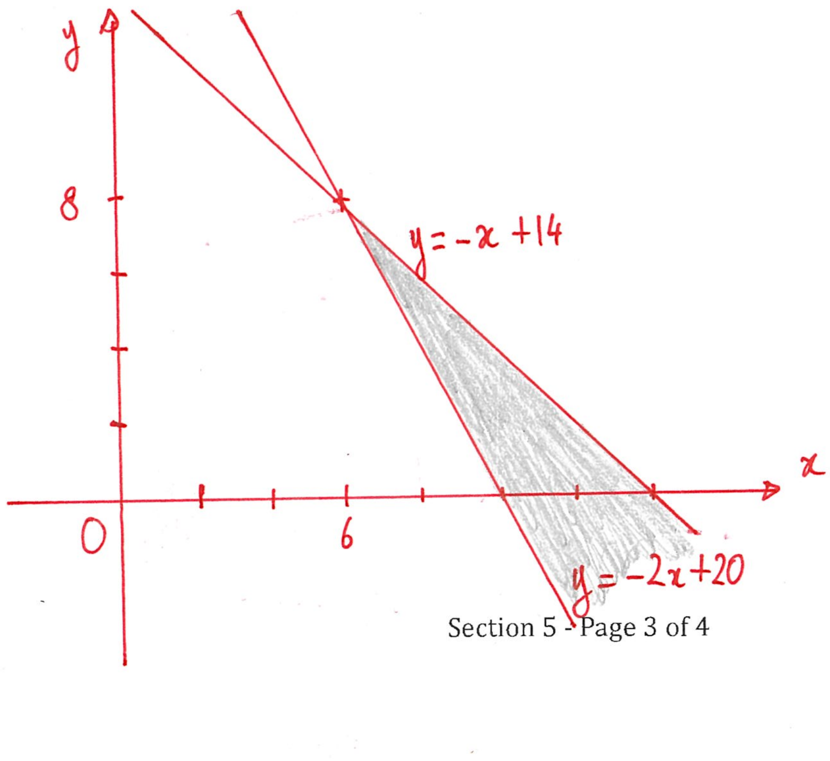
they intersect when

$-2x + 20 = -x + 14$

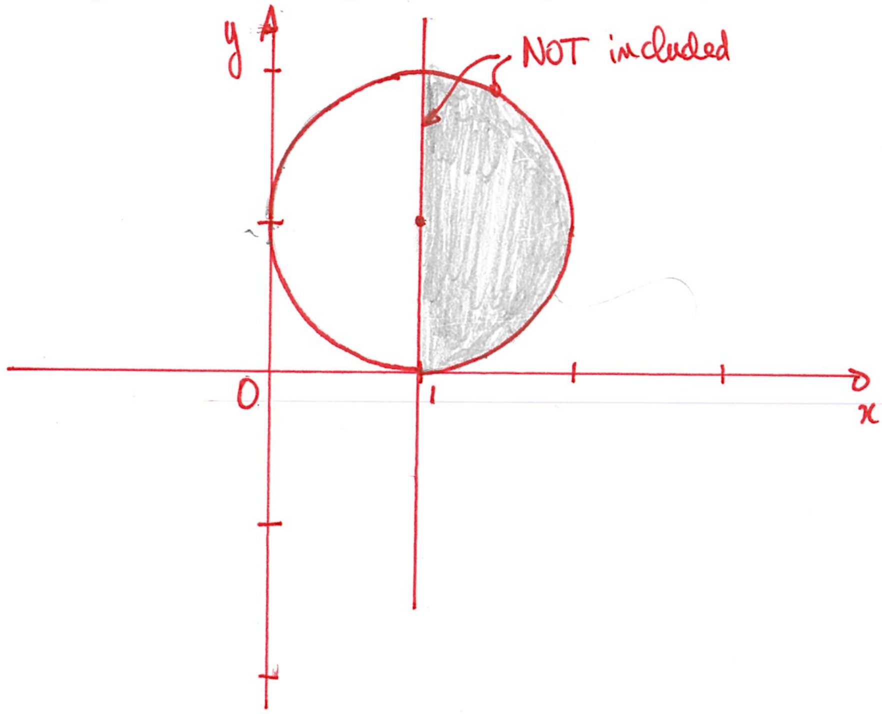
$\iff -x = -6$ i.e. $x = 6$

when $x = 6$, $y = -6 + 14 = 8$.

So intersection is $(6, 8)$



7 Sketch the region in the number plane defined by $(x-1)^2+(y-1)^2<1$ and $x>1$.



8 Sketch the region of the Cartesian plane bounded by the curves $y=x^2-4$ and $y=|x|+1$.