# VELOCITY AND ACCELERATION AS FUNCTIONS OF $x$

1 A particle moves in a straight line so that at time $t$ its displacement from a fixed origin is $x$ and its velocity is $v$. If its acceleration is given by $\ddot{x} = 4 + x$ and $v = 1$ when $x = 0$, find $v$ when $x = 1$.

3 At time $t$, the displacement of a particle moving in a straight line is $x$. If the acceleration is given by $\frac{d^2x}{dt^2} = 3 - 4x$ and the particle starts from rest at $x = 1$, find its velocity at any position. At what other point, if any, does the particle come to rest?

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**7** A particle moves in a straight line and its acceleration at any time is given by $\frac{d^2x}{dt^2} = \sin^2 x$. Find $\frac{dx}{dt}$ given that $\frac{dx}{dt} = 1$ when $x = 0$.

**10** The velocity of a particle is given by $v = 4 + x^2\ \text{m s}^{-1}$.

**(a)** Find the acceleration as a function of $x$.

**(b)** If initially $x = -2$ m, what is the displacement after $\frac{\pi}{4}$ seconds?

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**14** If $\frac{dx}{dt}=(3-x)^2$ and $x=2$ when $t=0$, find: **(a)** $x$ as a function of $t$ **(b)** $\frac{d^2x}{dt^2}$ as a function of $x$.

**15** A particle moves in a straight line. At time $t$ its displacement from a fixed origin is $x$. If $\dot{x}=x+3$:

**(a)** express $\ddot{x}$ in terms of $x$ **(b)** find $x$ when $t=1$, given that $x=-2$ when $t=0$.

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**16** The acceleration of a body moving under gravitational attraction towards a planet varies inversely as the square of its distance from the centre of the planet. This can be written as $\frac{d^2x}{dt^2} = -\frac{k}{x^2}$ where $x$ is the distance from the centre of the planet and $k$ is a constant. If the body starts from rest at a distance $a$ from the centre of the planet, show that its speed at $x$ (before it hits the planet) is given by $\frac{dx}{dt} = \sqrt{\frac{2k(a-x)}{ax}}$.

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**17** A particle is moving in a straight line with its acceleration as a function of $x$ given by $\ddot{x} = -e^{-2x}$. It is initially at the origin and travelling with a velocity of 1 metre per second.

**(a)** Show that $\dot{x} = e^{-x}$. **(b)** Hence show that $x = \log_e(t+1)$.

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**18** A particle is moving so that $\ddot{x} = 32x^3 + 48x^2 + 16x$. Initially $x = -2$ and the velocity $v$ is $-8$.

**(a)** Show that $v^2 = 16x^2(1 + x)^2$. **(b)** Hence, or otherwise, show that $-4t = \int \frac{1}{x(1+x)} dx$.

**(c)** It can be shown that for some constant $C$, $\log_e\left(1+\frac{1}{x}\right) = 4t + C$. Using this equation and the initial conditions, find $x$ as a function of $t$.

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**20** A body falls from rest so that its velocity $v$ metres per second after $t$ seconds is $v = 80(1 - e^{-0.4t})$.

**(a)** Show that the acceleration is proportional to $(80 - v)$.

**(b)** Calculate the distance fallen in the first five seconds.

**(c)** Calculate the distance fallen when $v = 60$.

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**21** A particle is brought to top speed by an acceleration that varies linearly with the distance travelled, i.e. $\ddot{x} = kx + C$ where $k$ and $C$ are constants. It starts from rest with an acceleration of $3\,\text{m}\,\text{s}^{-2}$ and reaches top speed in a distance of 160 metres. Find:

**(a)** the top speed **(b)** the speed when the particle has moved 80 metres.