CIRCLES

1 Find the equation of each of the following circles.

(a) centre $(3, 2)$, radius 4 units **(b)** centre $(-1, -4)$, radius 3 units **(c)** centre $(3, -3)$, radius $\sqrt{5}$ units

(d) centre $\left(-2, \frac{5}{2}\right)$, radius $\frac{7}{2}$ units **(e)** centre $\left(0, -\frac{3}{2}\right)$, radius 4 units **(f)** centre $(4, 0)$, radius 3 units

a) $(x-3)^2 + (y-2)^2 = 4^2 = 16$ b) $(x+1)^2 + (y+4)^2 = 3^2 = 9$

c) $(x-3)^2 + (y+\sqrt{3})^2 = 5$ d) $(x+2)^2 + \left(y-\frac{5}{2}\right)^2 = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$

e) $(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = 4^2 = 16$ or $x^2 + \left(y+\frac{3}{2}\right)^2 = 16$

f) $(x-4)^2 + y^2 = 9$

2 The equation of the circle with centre $(-4, 4)$ and radius 6 units is:

A $(x-4)^2 + (y-4)^2 = 36$ $\boxed{\textbf{B}}$ $(x+4)^2 + (y-4)^2 = 36$ **C** $(x-4)^2 + (y+4)^2 = 36$ **D** $x^2 + y^2 = 36$

3 Find the equation for each of the following circles.

(a) centre $(3, 2)$ and passing through the point $(5, -5)$
(b) centre $(-1, 4)$ and passing through the origin
(c) centre $(0, 0)$ and passing through the point $(-3, 4)$

a) Distance between centre and point $(5,-5) = \sqrt{(3-5)^2 + (2+5)^2} = \sqrt{53}$

So $(x-3)^2 + (y-2)^2 = 53$

b) Distance centre to origin $= \sqrt{(-1-0)^2 + (4-0)^2} = \sqrt{17}$

So $(x+1)^2 + (y-4)^2 = 17$

c) Distance from centre to $(-3, 4) = \sqrt{(0+3)^2 + (0-4)^2} = \sqrt{25} = 5$

So $(x+0)^2 + (y-0)^2 = 25$

OR $x^2 + y^2 = 25$

CIRCLES

4 Find the coordinates of the centre and the length of the radius for the following circles.

(a) $x^2 + y^2 - 6x + 4y - 3 = 0$ **(b)** $x^2 + y^2 + 4x + 2y - 4 = 0$

(e) $x^2 + y^2 - 5x + 3y - 1 = 0$ **(f)** $x^2 + y^2 + 4x + 2y - 5 = 0$

(g) $2x^2 + 2y^2 - 8x + 5y + 3 = 0$ **(h)** $3x^2 + 3y^2 + 9x - 4y - 24 = 0$

a) $x^2+y^2-6x+4y-3=0$

$\iff x^2-6x+y^2+4y=3$

$\iff (x-3)^2-9+(y+2)^2-4=3$

$\iff (x-3)^2+(y+2)^2=16=4^2$

So centre $(3,-2)$ radius 4

e) $\iff x^2-5x+y^2+3y=1$

$\iff \left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=1$

$\iff \left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{19}{2}$

So centre $\left(\frac{5}{2},\frac{-3}{2}\right)$ radius $\sqrt{\frac{19}{2}}$

g) $\iff x^2+y^2-4x+\frac{5}{2}y+\frac{3}{2}=0$

$\iff (x-2)^2-4+\left(y+\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2=-\frac{3}{2}$

$\iff (x-2)^2+\left(y+\frac{5}{4}\right)^2=\frac{65}{16}$

So centre $\left(2,-\frac{5}{4}\right)$ radius $\frac{\sqrt{65}}{4}$

b) $\iff x^2+4x+y^2+2y=4$

$\iff (x+2)^2-4+(y+1)^2-1=4$

$\iff (x+2)^2+(y+1)^2=9$

So centre $(-2,-1)$ and radius 3

f) $\iff x^2+4x+y^2+2y=5$

$\iff (x+2)^2-4+(y+1)^2-1=5$

$\iff (x+2)^2+(y+1)^2=10$

So centre $(-2,-1)$ radius $\sqrt{10}$

h) $\iff x^2+y^2+3x-\frac{4}{3}y-8=0$

$\iff \left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\left(y-\frac{4}{6}\right)^2-\left(\frac{4}{6}\right)^2=8$

$\iff \left(x+\frac{3}{2}\right)^2+\left(y-\frac{2}{3}\right)^2=8+\frac{9}{4}+\frac{4}{9}$

$\iff \left(x+\frac{3}{2}\right)^2+\left(y-\frac{2}{3}\right)^2=\frac{385}{36}$

So centre $\left(\frac{-3}{2},\frac{2}{3}\right)$ radius $\frac{\sqrt{385}}{6}$

5 Using the fact that the centre of a circle is the midpoint of a diameter, find the equation of the circle with the diameter endpoints given.

(a) (3, 4) and (9, −6) **(b)** (0, 0) and (5, −3) **(c)** (5, 8) and (−2, 3)

a) midpoint of (3, 4) and (9, −6) is $\left(\frac{3+9}{2}, \frac{4-6}{2}\right)$ which is $(6,-1)$

distance centre to (3,4) is $\sqrt{(3-6)^2+(4+1)^2} = \sqrt{9+25} = \sqrt{34}$

So the equation of this circle is $(x-6)^2+(y+1)^2 = 34$

b) midpoint of (0,0) and (5,−3) is $\left(\frac{5}{2}, \frac{-3}{2}\right)$

distance centre to $(5/2, -3/2)$ is $\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{3}{2}\right)^2} = \frac{\sqrt{34}}{2}$

So the equation of this circle is $\left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2 = \frac{34}{4} = \frac{17}{2}$

c) midpoint of (5,8) and (−2,3) is $\left(\frac{5-2}{2}, \frac{8+3}{2}\right)$ or $\left(\frac{3}{2}, \frac{11}{2}\right)$

distance centre to (5, 8) is $\sqrt{\left(5-\frac{3}{2}\right)^2+\left(8-\frac{11}{2}\right)^2} = \sqrt{\frac{49}{4}+\frac{25}{4}} = \frac{\sqrt{74}}{2}$

So the equation of this circle is:

$$(x-5)^2+(y-8)^2 = \frac{74}{4} = \frac{37}{2}$$

6 For the equation $x^2 + y^2 - 6x + 2y + 10 = 0$, indicate whether each statement is correct or incorrect.

(a) centre (3, −1), radius = 1 **(b)** centre (−3, 1), radius = 0

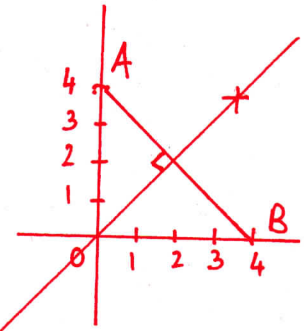
(c) centre (3, −1), radius = $2\sqrt{5}$ $\boxed{\textbf{(d)}}$ centre (3, −1), radius = 0

$\Leftrightarrow x^2-6x+y^2+2y = -10$ $\Leftrightarrow (x-3)^2-9+(y+1)^2-1 = -10$

$\Leftrightarrow (x-3)^2+(y+1)^2 = 0$

CIRCLES

12 Find the equation of the circle that touches the x-axis at $(4, 0)$ and the y-axis at $(0, 4)$.



The centre must be equidistant from those two points, $\therefore$ must be on the line $y = x$ as it bisects the segment [AB]

For the circle to only touch (and not cross) the axis, that means that the centre must be at $(4,4)$

So $(x-4)^2 + (y-4)^2 = 4^2 = 16$

13 Show that the point $(4, -3)$ is not on the circle $x^2 + y^2 - 5x + 3y + 2 = 0$. Determine whether the point is inside or outside the circle.

Substituting $x = 4$ and $y = -3$ in the LHS gives:

$4^2 + (-3)^2 - 5 \times 4 + 3 \times (-3) + 2 = 16 + 9 - 20 - 9 + 2 = -2$

which is $\neq$ of 0. $\therefore$ $(4,-3)$ does not belong to the circle.

$x^2 + y^2 - 5x + 3y + 2 = 0 \iff (x - 5/2)^2 - \frac{25}{4} + \left(y + \frac{3}{2}\right)^2 - \frac{9}{4} = -2$

$\iff \left(x - 5/2\right)^2 + \left(y + 3/2\right)^2 = 13/2 = \left(\sqrt{\frac{13}{2}}\right)^2$ So centre $\left(\frac{5}{2}, -\frac{3}{2}\right)$ radius $\sqrt{\frac{13}{2}}$

Distance from centre to $(4,-3)$ $= \sqrt{\left(\frac{5}{2} - 4\right)^2 + \left(-\frac{3}{2} + 3\right)^2} = \sqrt{\frac{9}{4} + \frac{9}{4}} = \frac{\sqrt{18}}{2} = \frac{3\sqrt{2}}{2}$

As $\frac{3\sqrt{2}}{2} \sim 2.12$ is less than $\sqrt{\frac{13}{2}} \sim 2.54$ that means the point is inside.

14 Determine whether the origin is inside or outside the circle $x^2 + y^2 - 4x - y + 1 = 0$.

$\iff x^2 - 4x + y^2 - y = -1 \iff (x-2)^2 - 4 + (y - 1/2)^2 - \frac{1}{4} = -1$

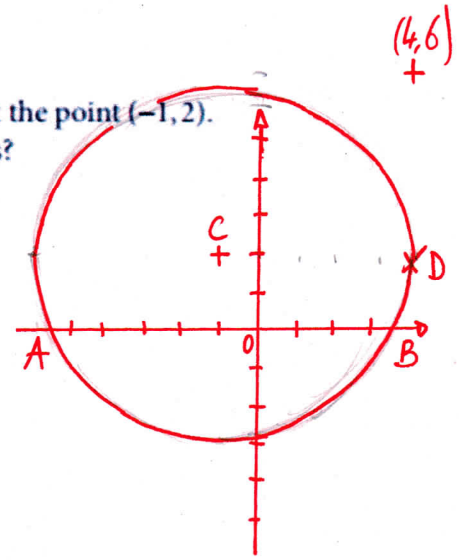
$\iff (x-2)^2 + (y - 1/2)^2 = \frac{13}{4} = \left(\frac{\sqrt{13}}{2}\right)^2$

Distance from origin to centre is $\sqrt{2^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{17}}{2}$

whereas the radius is $\sqrt{13}/2$. But $\frac{\sqrt{17}}{2} > \frac{\sqrt{13}}{2}$

As the origin is further away from the centre than the length of the radius that means it's <u>outside</u> of the circle.

15 **(a)** Find the equation of a circle with a radius of 5 units and its centre at the point $(-1, 2)$.
(b) What is the length of the intercept cut off by this circle on the x-axis?
(c) Find the length of the tangent to this circle from the point $(4, 6)$.



a) $(x+1)^2+(y-2)^2=25$

b) when $y=0$, then $(x+1)^2+(0-2)^2=25$

$\Leftrightarrow (x+1)^2=25-4=21$

so $x+1=\pm\sqrt{21}$ so $x=-1\pm\sqrt{21}$

So the distance AB between the two intersects is

$(-1+\sqrt{21})-(-1-\sqrt{21})=2\sqrt{21}$

c) The point D shown on the diagram has for coordinates $(4,2)$

Therefore, its x-coordinate is the same than for point $(4,6)$

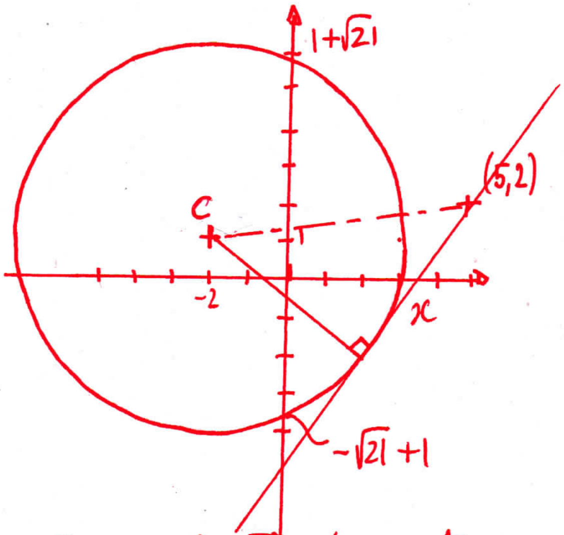
Therefore the tangent to the circle that passes through $(4,6)$ is a vertical line.

Distance between $D(4,2)$ and $(4,6)$ is 4 units long.

16 The equation of a circle is $x^2 + y^2 + 4x - 2y - 20 = 0$. Find:

(a) the length of the tangent to this circle from the point $(5, 2)$

(b) the length of the intercept on the y-axis.



$\Leftrightarrow \quad x^2 + 4x + y^2 - 2y = 20$

$\Leftrightarrow \quad (x+2)^2 - 4 + (y-1)^2 - 1 = 20$

$\Leftrightarrow \quad (x+2)^2 + (y-1)^2 = 25 = 5^2$

So centre $(-2, 1)$, radius 5

a) We use Pythagoras Theorem, as the radius and the tangent to the circle are perpendicular.

So $[\text{Distance centre to } (5,2)]^2 = (\text{radius})^2 + x^2$

so $x^2 = \left[\sqrt{(5-(-2))^2 + (2-1)^2}\right]^2 - 25$

$x^2 = \left[\sqrt{7^2+1^2}\right]^2 - 25 = 50 - 25 = 25$ so $x = 5$

b) When $x = 0$ the equation becomes $(0+2)^2 + (y-1)^2 = 25$

$\Leftrightarrow \quad (y-1)^2 = 21$ so $y - 1 = \pm\sqrt{21}$ $\quad y = 1 \pm \sqrt{21}$

So the length l of the intercept on the y-axis is

$l = (1+\sqrt{21}) - (1-\sqrt{21}) = 2\sqrt{21}$

18 The coordinates of two points A and B are $(-1,3)$ and $(5,7)$. Find:

(a) the coordinates of the midpoint of AB
(b) the equation of the circle of which AB is a diameter
(c) the coordinates of the intersection points of the circle with the y-axis.

a) mid point of AB has for coordinates $\left(\frac{-1+5}{2}, \frac{3+7}{2}\right)$ or $(2,5)$

b) Distance AB is $\sqrt{(-1-5)^2+(3-7)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

so the radius of the circle is $\sqrt{13}$

The equation of this circle is $(x-2)^2+(y-5)^2 = 13$

or $x^2 - 4x + 4 + y^2 - 10y + 25 = 13$

$\Leftrightarrow \quad x^2 - 4x + y^2 - 10y + 16 = 0$

c) When $x = 0$, the equation becomes: $(0-2)^2+(y-5)^2 = 13$

or $4 + (y-5)^2 = 13 \quad \Leftrightarrow \quad (y-5)^2 = 9$

so $y - 5 = \pm 3 \qquad y = 5 \pm 3$

So either $y = 2$ or $y = 8$

The coordinates of the intersection points of the circle with the x-axis are $(0,2)$ and $(0,8)$

19 **(a)** Find the coordinates of the centre and the length of the radius for the circle $x^2 + y^2 - 4x - 8y - 5 = 0$.
(b) The point $(3, 2)$ is the midpoint of a chord of this circle. Find the distance of the chord from the centre and the length of the chord.

a) $\Leftrightarrow \quad x^2 - 4x + y^2 - 8y = 5$

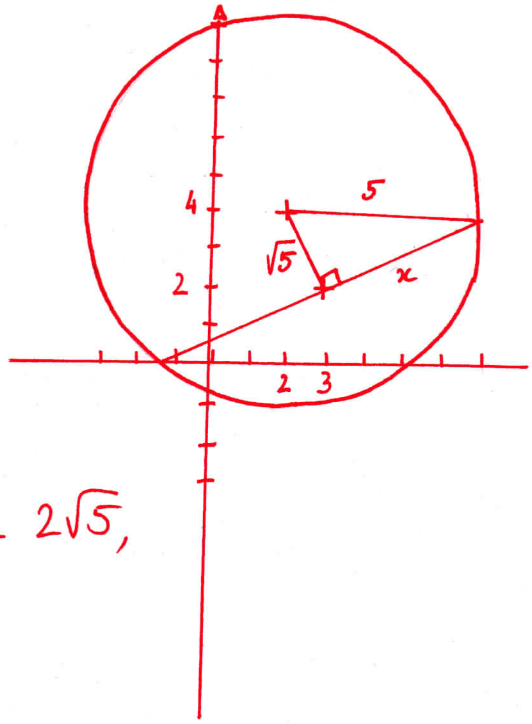
$\Leftrightarrow \quad (x-2)^2 - 4 + (y-4)^2 - 16 = 5$

$\Leftrightarrow \quad (x-2)^2 + (y-4)^2 = 25 = 5^2$

So centre $(2, 4)$, radius 5

b) Distance centre to $(3,2) = \sqrt{(3-2)^2 + (2-4)^2} = \sqrt{1+4} = \sqrt{5}$

Then we use Pythagoras theorem:



$5^2 = (\sqrt{5})^2 + x^2$

so $x^2 = 25 - 5 = 20$

so $x = \sqrt{20} = 2\sqrt{5}$

So the length of the chord is twice $2\sqrt{5}$,

i.e $4\sqrt{5}$