TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Express each of the following as sums or differences:

2 $2\sin 4\theta \cos 2\theta$	**3** $2\cos 3A \cos 5A$	**4** $\cos 4A \sin 2A$
5 $\sin(\theta+\alpha)\cos(\theta-\alpha)$	**6** $2\cos(45°+A)\sin(45°-A)$	**7** $\cos(2\theta+\alpha)\cos(2\theta-\alpha)$

2) $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$, therefore

$2\sin 4\theta \cos 2\theta = 2\times\frac{1}{2}\left[\sin(4\theta+2\theta) + \sin(4\theta-2\theta)\right] = \sin 6\theta + \sin 2\theta$

3) $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$, therefore

$2\cos 3A \cos 5A = 2\times\frac{1}{2}\left[\cos(3A+5A) + \cos(5A-3A)\right] = \cos 8A + \cos 2A$

4) $\cos 4A \sin 2A = \frac{1}{2}\left[\sin(4A+2A) + \sin(2A-4A)\right] = \frac{1}{2}\left[\sin 6A - \sin 2A\right]$

5) $\sin(\theta+\alpha)\cos(\theta-\alpha) = \frac{1}{2}\left[\sin\left[(\theta+\alpha)+(\theta-\alpha)\right] + \sin\left[(\theta+\alpha)-(\theta-\alpha)\right]\right]$

$= \frac{1}{2}\left[\sin(2\theta) + \sin(2\alpha)\right]$

6) $2\sin(45-A)\cos(45+A) = 2\times\frac{1}{2}\left[\sin\left[(45-A)+(45+A)\right] + \sin\left[(45-A)-(45+A)\right]\right]$

$= \sin 90 + \sin(-2A) = 1 - \sin 2A.$

7) $\cos(2\theta+\alpha)\cos(2\theta-\alpha) = \frac{1}{2}\left[\cos\left[(2\theta+\alpha)+(2\theta-\alpha)\right] + \cos\left[(2\theta+\alpha)-(2\theta-\alpha)\right]\right]$

$= \frac{1}{2}\left[\cos 4\theta + \cos 2\alpha\right]$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

14 $\cos 75° \cos 45°$ **15** $2\sin(\theta+\phi)\cos(\theta-\phi)$ **16** $\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

17 $\sin 100° \sin 130°$ **18** $2\sin 3\theta \cos\theta$ **19** $\cos(\theta+2\phi)\sin(2\theta+\phi)$

14) $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$, therefore:

$\cos 75 \cos 45 = \frac{1}{2}\left[\cos(75+45) + \cos(75-45)\right] = \frac{1}{2}\left[\cos(120) + \cos(30)\right] = \frac{1}{2}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}\right]$

$\_\_\_\_ = -\frac{1}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}-1}{4}$

15) $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$ therefore:

$2\sin(\theta+\phi)\cos(\theta - \phi) = 2 \times \frac{1}{2}\left[\sin[(\theta+\phi)+(\theta-\phi)] + \sin[(\theta+\phi)-(\theta-\phi)]\right]$

$\_\_\_\_ = \sin(2\theta) + \sin(2\phi)$

16) $\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \frac{1}{2}\left[\sin\left[\left(\frac{A+B}{2}\right)+\left(\frac{A-B}{2}\right)\right] + \sin\left[\left(\frac{A+B}{2}\right)-\left(\frac{A-B}{2}\right)\right]\right]$

$\_\_\_\_ = \frac{1}{2}\left[\sin A + \sin B\right]$

17) $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$ therefore

$\sin 100 \sin 130 = \frac{1}{2}\left[\cos(100-130) - \cos(100+130)\right]$

$\_\_\_\_ = \frac{1}{2}\left[\cos 30 - \cos 230\right] = \frac{\sqrt{3}}{4} - \frac{\cos 230}{2}$

18) $2\sin 3\theta \cos\theta = 2 \times \frac{1}{2}\left[\sin(4\theta) + \sin(2\theta)\right] = \sin 4\theta + \sin 2\theta$

19) $\sin(2\theta+\phi)\cos(\theta+2\phi) = \frac{1}{2}\left[\sin(3\theta+3\phi) + \sin(\theta-\phi)\right]$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Express the following as products:

23 $\sin 3x - \sin x$

24 $\sin(x+\alpha) - \sin x$

25 $\cos(x+h) - \cos x$

26 $\sin(\theta+\alpha) + \sin(\theta-\alpha)$

27 $\cos\left(\frac{\theta+\alpha}{2}\right) + \cos\left(\frac{\theta-\alpha}{2}\right)$

28 $\cos(A+B+C) - \cos(A-B+C)$

23) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ therefore:

$\sin 3x - \sin x = 2\cos 2x \sin x$

24) $\sin(x+\alpha) - \sin x = 2\cos\left(\frac{2x+\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right) = 2\cos\left(x+\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right)$

25) $\cos\phi - \cos\theta = 2\sin\left(\frac{\phi+\theta}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$ therefore:

$\cos(x+h) - \cos x = 2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right) = 2\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$

26) $\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$ therefore:

$\sin(\theta+\alpha) + \sin(\theta-\alpha) = 2\sin\theta\cos\alpha$

27) $\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$ therefore:

$\cos\left(\frac{\theta+\alpha}{2}\right) + \cos\left(\frac{\theta-\alpha}{2}\right) = 2\cos\left(\frac{\frac{\theta+\alpha}{2}+\frac{\theta-\alpha}{2}}{2}\right)\cos\left(\frac{\frac{\theta+\alpha}{2}-\frac{\theta-\alpha}{2}}{2}\right) = 2\cos\frac{\theta}{2}\cos\frac{\alpha}{2}$

28) $\cos(A+B+C) - \cos(A-B+C) = -2\sin\left(\frac{(A+B+C)+(A-B+C)}{2}\right)\sin\left(\frac{(A+B+C)-(A-B+C)}{2}\right)$

$= -2\sin(A+C)\sin B.$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

36 $\sin(2A+2B)-\sin(2A-2B)$ **37** $\sin 165° - \sin 105°$

39 $\cos 75° - \cos 45°$ **40** $\sin 50° + \cos 20°$ **41** $\sin(A-B)-\sin A$

36) $\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$ therefore:

$\sin(2A+2B)-\sin(2A-2B) = 2\cos\left[\frac{(2A+2B)+(2A-2B)}{2}\right]\sin\left[\frac{(2A+2B)-(2A-2B)}{2}\right]$

$= 2\cos(2A)\sin(2B)$

37) $\sin 165 - \sin 105 = 2\cos\left(\frac{165+105}{2}\right)\sin\left(\frac{165-105}{2}\right)$

$= 2\cos 135 \sin 30 = 2\times\left(-\frac{\sqrt{2}}{2}\right)\times\frac{1}{2} = -\sqrt{2}/2$

39) $\cos 75 - \cos 45 = 2\sin\left(\frac{75+45}{2}\right)\sin\left(\frac{75-45}{2}\right)$

$= 2\sin 60 \sin 15 = 2\frac{\sqrt{3}}{2}\sin 15 = \sqrt{3}\sin 15$

$\cos 30 = 1 - 2\sin^2 15$ so $\sin 15 = \sqrt{\frac{1-\cos 30}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}}$

40) $\sin 50 + \cos 20 = \sin 50 + \sin(90-20) = \sin 50 + \sin 70$

$= 2\sin\left(\frac{50+70}{2}\right)\cos\left(\frac{50-70}{2}\right) = 2\sin 60\cos 10 = \sqrt{3}\cos 10$

41) $\sin(A-B) - \sin A = 2\cos\left(\frac{(A+B)+A}{2}\right)\sin\left(\frac{(A+B)-A}{2}\right)$

$= 2\cos\left[A+\frac{B}{2}\right]\sin\left(\frac{B}{2}\right)$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Prove the following results.

43 $\dfrac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \tan\theta$

44 $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\dfrac{x+y}{2}\right)$

45 $\dfrac{\cos x - \cos 3x}{\sin x - \sin 3x} = -\tan 2x$

46 $\dfrac{\sin 2A - \sin 2B}{\sin 2A + \sin 2B} = \dfrac{\tan(A-B)}{\tan(A+B)}$

43) $\dfrac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \dfrac{2\cos 4\theta \sin\theta}{2\cos 4\theta \cos\theta} = \tan\theta$

44) $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \dfrac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \tan\left(\dfrac{x+y}{2}\right)$

45) $\dfrac{\cos x - \cos 3x}{\sin x - \sin 3x} = \dfrac{2\sin 2x \sin(-x)}{2\cos 2x \sin(+x)} = \tan 2x \times \left(\dfrac{-\sin x}{\sin x}\right) = -\tan 2x$

46) $\dfrac{\sin 2A - \sin 2B}{\sin 2A + \sin 2B} = \dfrac{2\cos(A+B)\sin(A-B)}{2\sin(A+B)\cos(A-B)}$

$\_\_\_\_\_\_ = \dfrac{\sin(A-B)\cos(A+B)}{\cos(A-B)\sin(A+B)}$

$\_\_\_\_\_\_ = \tan(A-B) \times \dfrac{1}{\dfrac{\sin(A+B)}{\cos(A+B)}}$

$\_\_\_\_\_\_ = \dfrac{\tan(A-B)}{\tan(A+B)}$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Prove the following results.

54 $2\cos 37.5^\circ \sin 7.5^\circ = \dfrac{\sqrt{2}-1}{2}$

55 $\sin 25^\circ \sin 35^\circ - \sin 20^\circ \sin 10^\circ = \dfrac{\sqrt{3}-1}{4}$

56 $\sin 10^\circ + \cos 40^\circ = \sin 70^\circ$

57 $\dfrac{\sin 48^\circ + \sin 12^\circ}{\cos 48^\circ + \cos 12^\circ} = \dfrac{\sqrt{3}}{3}$

54) $2\cos(37.5)\sin(7.5) = \dfrac{2}{2}\left[\sin(37.5+7.5) + \sin(7.5-37.5)\right]$

$\underline{\hspace{2cm}} = \sin 45 - \sin 30 = \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} = \dfrac{\sqrt{2}-1}{2}$

55) $\sin 25 \sin 35 - \sin 20 \sin 10 = \dfrac{1}{2}\left[\cos(-10) - \cos(60)\right] - \dfrac{1}{2}\left[\cos(10) - \cos 30\right]$

$\underline{\hspace{2cm}} = \dfrac{1}{2}\cos 10 - \dfrac{1}{2}\cos 10 - \dfrac{1}{4} + \dfrac{\sqrt{3}}{4} = \dfrac{\sqrt{3}-1}{4}$

56) $\sin 10 + \cos 40 = \cos(90-10) + \cos 40 = \cos 80 + \cos 40$

$\underline{\hspace{2cm}} = 2\cos\left(\dfrac{80+40}{2}\right)\cos\left(\dfrac{80-40}{2}\right) = 2\cos 60 \cos 20 = \cos 20$

$= \sin 70$

57) $\dfrac{\sin 48 + \sin 12}{\cos 48 + \cos 12} = \dfrac{2\sin\left(\dfrac{60}{2}\right)\cos\left(\dfrac{48-12}{2}\right)}{2\cos\left(\dfrac{60}{2}\right)\cos\left(\dfrac{48-12}{2}\right)} = \dfrac{\sin 30}{\cos 30} = \tan 30$

$\underline{\hspace{2cm}} = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Prove the following results.

60 $\dfrac{\sin\theta + \sin 7\theta}{\sin 3\theta + \sin 5\theta} = 2\cos 2\theta - 1$

61 $\dfrac{\sin(\theta+\phi) - \sin(\theta-\phi)}{\cos(\theta+\phi) - \cos(\theta-\phi)} = -\cot\theta$

60) $$\frac{\sin\theta + \sin 7\theta}{\sin 3\theta + \sin 5\theta} = \frac{2\sin\left(\frac{\theta+7\theta}{2}\right)\cos\left(\frac{\theta-7\theta}{2}\right)}{2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)} = \frac{\sin 4\theta\cos(-3\theta)}{\sin 4\theta\cos(-\theta)}$$

$$\_\_\_ = \frac{\cos 3\theta}{\cos\theta} = \frac{\cos(\theta+2\theta)}{\cos\theta} = \frac{\cos\theta\cos 2\theta - \sin\theta\sin 2\theta}{\cos\theta}$$

$$\_\_\_ = \cos 2\theta - \frac{\sin\theta}{\cos\theta} \times 2\sin\theta\cos\theta = \cos 2\theta - 2\sin^2\theta$$

$$\_\_\_ = \cos 2\theta + [\cos 2\theta - 1] = 2\cos 2\theta - 1$$

61) $$\frac{\sin(\theta+\phi) - \sin(\theta-\phi)}{\cos(\theta+\phi) - \cos(\theta-\phi)} = \frac{2\cos\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\sin\left[\frac{(\theta+\phi)-(\theta-\phi)}{2}\right]}{2\sin\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\sin\left[\frac{(\theta-\phi)-(\theta+\phi)}{2}\right]}$$

$$\_\_\_ = \frac{\cos\theta\ \sin\phi}{\sin\theta\ \sin(-\phi)}$$

$$\_\_\_ = \frac{\cos\theta}{\sin\theta} \times \frac{\sin\phi}{(-\sin\phi)}$$

$$\_\_\_ = -\cot\theta$$

TRIGONOMETRIC PRODUCTS AS SUMS OR DIFFERENCES

Prove the following results.

65 If $\alpha+\beta+\gamma=\pi$, show that $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=4\sin\alpha\sin\beta\sin\gamma$.

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 2\sin\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right) + \sin 2\gamma$$

$$= 2\sin(\alpha+\beta)\cos(\alpha-\beta) + \sin[2(\pi-\alpha-\beta)]$$

$$= 2\sin(\alpha+\beta)\cos(\alpha-\beta) - \sin[2(\alpha+\beta)]$$

$$= 2\sin(\alpha+\beta)\cos(\alpha-\beta) - 2\sin(\alpha+\beta)\cos(\alpha+\beta)$$

$$= 2\sin(\alpha+\beta)\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$= 2\sin(\alpha+\beta) \times \left[-2\sin\left[\frac{(\alpha-\beta)+(\alpha+\beta)}{2}\right]\sin\left[\frac{(\alpha-\beta)-(\alpha+\beta)}{2}\right]\right]$$

$$= -4\sin(\alpha+\beta)\sin\alpha\sin(-\beta)$$

$$= 4\sin\alpha\sin\beta\sin(\alpha+\beta).$$

But $\alpha+\beta=\pi-\gamma$ so $\sin(\alpha+\beta)=\sin(\pi-\gamma)=\sin\gamma$

$$\therefore \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha\sin\beta\sin\gamma$$