RECURRENCE RELATIONS

1 **(a)** Differentiate $\sin x \cos^{n-1} x$ with respect to x to show that if $I_n = \int \cos^n x \, dx$, then:

$$I_n = \frac{1}{n} \sin x \cos^{n-1} x + \frac{n-1}{n} I_{n-2}$$

(b) Hence evaluate: **(i)** $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$ **(iii)** $\int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x \, dx$

RECURRENCE RELATIONS

2 Show that $\int x^n e^x \, dx = x^n e^x - n\int x^{n-1} e^x \, dx$ and hence find: $\int x^3 e^x \, dx$

3 **(a)** Find the derivative of $x^n \log_e x$.

(b) Hence find (correct to three decimal places) the value of:

(i) $\int_1^2 x^2 \log_e x \, dx$ **(ii)** $\int_1^2 x^3 \log_e x \, dx$

RECURRENCE RELATIONS

4 (a) Given that $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$, prove that $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$ where $n \geq 2$ is an integer.

(b) Hence evaluate: $\int_0^{\frac{\pi}{2}} \cos^5 x\,dx$

5 **(a)** Show that: $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

(b) Hence evaluate: $\int_0^{\frac{\pi}{6}} \sin^4 x\,dx$

RECURRENCE RELATIONS

6 **(a)** If $I_n = \int \sec^n x\,dx$ show that: $I_n = \frac{1}{n-1}(\sec^{n-2} x \tan x) + \frac{n-2}{n-1} I_{n-2}$

(b) Hence evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^4 x\,dx$

RECURRENCE RELATIONS

8 **(a)** Given that $I_n = \int_0^1 x^{2n-1} e^{x^2}\, dx$ for each integer $n \geq 1$, show that: $I_n = \frac{e}{2} - (n-1)I_{n-1}$

(b) Hence, or otherwise, calculate I_2.

RECURRENCE RELATIONS

10 **(a)** Show that: $\int e^{ax}\sin x\,dx = \frac{1}{a^2+1}e^{ax}(a\sin x - \cos x)$

(b) Hence find: **(i)** $\int e^{x}\sin x\,dx$ **(ii)** $\int e^{3x}\sin x\,dx$

11 If $I_n = \int_0^x \frac{t^n}{1+t}\,dt$ show that: $I_n + I_{n-1} = \frac{x^n}{n}$