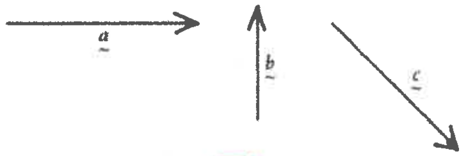
INTRODUCTION TO VECTORS

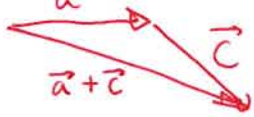
1 Given three vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, as shown, construct the following:

(a) $\underset{\sim}{a}+\underset{\sim}{c}$ **(b)** $\underset{\sim}{c}+\underset{\sim}{b}$

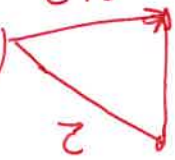
(c) $\underset{\sim}{a}-\underset{\sim}{b}$ **(d)** $\underset{\sim}{c}-\underset{\sim}{a}$



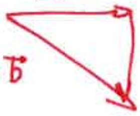
a)



b) $\vec{c}+b$



c) $\vec{a}-\vec{b}$

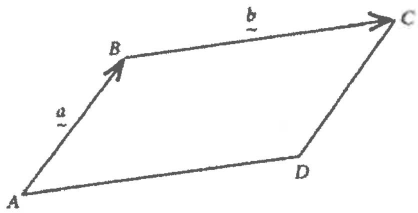


d) $\vec{c}-\vec{a}$

2 *ABCD* is a parallelogram. If $\overrightarrow{AB}=\underset{\sim}{a}$ and $\overrightarrow{BC}=\underset{\sim}{b}$, express each of the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(a) $\overrightarrow{CD}$ **(b)** $\overrightarrow{AD}$

(c) $\overrightarrow{CA}$ **(d)** $\overrightarrow{DB}$



a) $\vec{CD} = -\vec{AB} = -\vec{a}$

b) $\vec{AD} = \vec{BC} = \vec{b}$

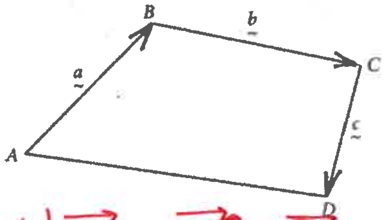
c) $\vec{CA} = \vec{CB} + \vec{BA} = -\vec{b} - \vec{a}$

d) $\vec{DB} = \vec{DA} + \vec{AB} = -\vec{b} + \vec{a}$

3 *ABCD* is a quadrilateral. If $\overrightarrow{AB}=\underset{\sim}{a}$, $\overrightarrow{BC}=\underset{\sim}{b}$ and $\overrightarrow{CD}=\underset{\sim}{c}$, express each of the following vectors in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

(a) $\overrightarrow{AC}$ **(b)** $\overrightarrow{AD}$

(c) $\overrightarrow{DA}$ **(d)** $\overrightarrow{DB}$



a) $\vec{AC} = \vec{AB} + \vec{BC} = \vec{a} + \vec{b}$

b) $\vec{AD} = \vec{AB} + \vec{BC} + \vec{AD} = \vec{a} + \vec{b} + \vec{c}$

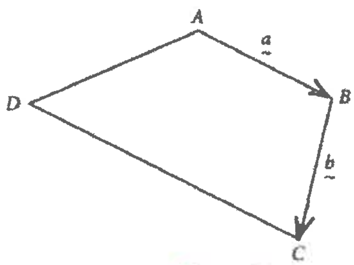
c) $\vec{DA} = -\vec{AD} = -(\vec{a} + \vec{b} + \vec{c}) = -\vec{a} - \vec{b} - \vec{c}$

d) $\vec{DB} = \vec{DC} + \vec{CB}$
$= -\vec{c} - \vec{b}$

4 *ABCD* is a trapezium with $\overrightarrow{DC}$ parallel to $\overrightarrow{AB}$ and one-and-a-half times the length of $\overrightarrow{AB}$. If $\overrightarrow{AB}=\underset{\sim}{a}$ and $\overrightarrow{BC}=\underset{\sim}{b}$, express each of the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(a) $\overrightarrow{CD}$ **(b)** $\overrightarrow{CA}$

(c) $\overrightarrow{AD}$ **(d)** $\overrightarrow{DB}$



a) $\vec{CD} = 1.5 \times \vec{BA} = -1.5\vec{a}$

b) $\vec{CA} = \vec{CB} + \vec{BA} = -\vec{b} - \vec{a}$

c) $\vec{AD} = \vec{AB} + \vec{BC} + \vec{CD}$

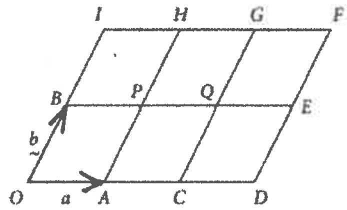
$\vec{AD} = \vec{a} + \vec{b} - 1.5\vec{a}$

$\vec{AD} = -0.5\vec{a} + \vec{b}$

d) $\vec{DB} = \vec{DA} + \vec{AB}$

$\vec{DB} = -\vec{AD} + \vec{a} = -(-0.5\vec{a} + \vec{b}) + \vec{a} = 1.5\vec{a} - \vec{b}$

5 If all the short line segments shown are the same length, express the following in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



(a) $\overrightarrow{OP}$ (b) $\overrightarrow{OG}$ (c) $\overrightarrow{OQ}$ (d) $\overrightarrow{CE}$
(e) $\overrightarrow{AB}$ (f) $\overrightarrow{DI}$ (g) $\overrightarrow{FQ}$ (h) $\overrightarrow{DE}+\overrightarrow{EO}$

a) $\overrightarrow{OP} = \vec{a} + \vec{b}$ b) $\overrightarrow{OG} = 2(\vec{a}+\vec{b})$

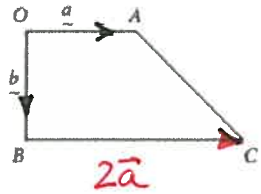
c) $\overrightarrow{OQ} = 2\vec{a} + \vec{b}$ d) $\overrightarrow{CE} = \overrightarrow{OP} = \vec{a} + \vec{b}$ e) $\overrightarrow{AB} = \vec{b} - \vec{a}$

f) $\overrightarrow{DI} = \overrightarrow{DO} + \overrightarrow{OI} = -3\vec{a} + 2\vec{b}$

g) $\overrightarrow{FQ} = \overrightarrow{PO} = -\overrightarrow{OP} = -(\vec{a}+\vec{b}) = -\vec{a} - \vec{b}$

h) $\overrightarrow{DE} + \overrightarrow{EO} = \overrightarrow{DO} = -3\vec{a}$

6 $\overrightarrow{BC}$ is parallel to $\overrightarrow{OA}$ and twice its length. Express the following in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

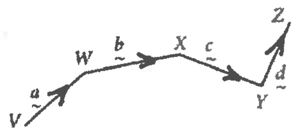


(a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\vec{a} - \vec{b}$

b) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC} = -\vec{a} + \vec{b} + 2\vec{a} = \vec{a} + \vec{b}$

7 From the diagram, find the following in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$ and $\underset{\sim}{d}$.



(a) $\overrightarrow{VY}$ (b) $\overrightarrow{VZ}$ (c) $\overrightarrow{WZ}$

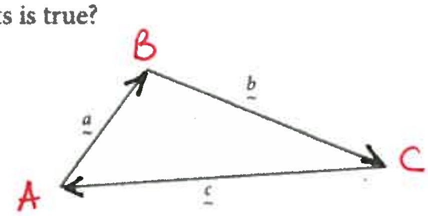
a) $\overrightarrow{VY} = \vec{a} + \vec{b} + \vec{c}$ b) $\overrightarrow{VZ} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

c) $\overrightarrow{WZ} = \vec{b} + \vec{c} + \vec{d}$

8 In ΔABC, $\overrightarrow{AB} = \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{b}$ and $\overrightarrow{CA} = \underset{\sim}{c}$. Which *one* of the following statements is true?

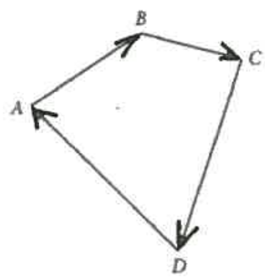
A $\underset{\sim}{a}+\underset{\sim}{c}=\underset{\sim}{b}$ $\boxed{\textbf{B}}$ $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{0}$

C $\underset{\sim}{a}+\underset{\sim}{b}-\underset{\sim}{c}=\underset{\sim}{0}$ **D** $\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{a}$

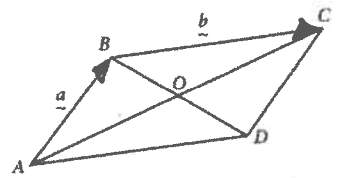


9 In the quadrilateral $ABCD$, which *one* of the following statements is true?

A $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{CD}+\overrightarrow{DA}$ NO
B $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{CD}-\overrightarrow{DA}$ NO
C $\overrightarrow{AB}-\overrightarrow{BC}=\overrightarrow{CD}-\overrightarrow{DA}$ NO
D $\overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CD}-\overrightarrow{DA}$ YES



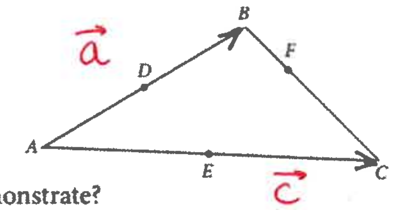
10 In the parallelogram $ABCD$ shown, the point of intersection of the diagonals is O, where O is the midpoint of both $\overrightarrow{AC}$ and $\overrightarrow{BD}$.



The vector $\overrightarrow{OC}$ is equal to:

A $\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b})$ $\quad$ $\boxed{\text{B}}$ $\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$ $\quad$ C $\frac{1}{2}\underset{\sim}{a}-\underset{\sim}{b}$ $\quad$ D $\frac{1}{2}\underset{\sim}{b}-\underset{\sim}{a}$

11 ΔABC is a triangle with $\overrightarrow{AB}=\underset{\sim}{a}$ and $\overrightarrow{AC}=\underset{\sim}{c}$. D and E are the midpoints of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively. F is a point on $\overrightarrow{BC}$ such that $\overrightarrow{FC}=2\times\overrightarrow{BF}$.



(a) Express the vectors $\overrightarrow{BC}$ and $\overrightarrow{DE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(b) Compare the vectors $\overrightarrow{BC}$ and $\overrightarrow{DE}$.

(c) What geometric property of a triangle does the answer to part (b) demonstrate?

(d) Express the vectors $\overrightarrow{BF}$ and $\overrightarrow{FC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(e) Show that $\overrightarrow{AF}=\frac{1}{3}(2\underset{\sim}{a}+\underset{\sim}{c})$.

a) $\vec{BC} = \vec{BA} + \vec{AC} = -\vec{a} + \vec{c}$ $\qquad$ $\vec{DE} = \vec{DA} + \vec{AE} = -\frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$

So $\vec{DE} = \frac{1}{2}\left[-\vec{a} + \vec{c}\right] = \frac{1}{2}\vec{BC}$

b) $\vec{BC} = 2\vec{DE}$

c) line BC is parallel to line DE and the segment BC is twice as long as the segment DE.

$\therefore$ triangles ADE and ABC are similar.

d) $\vec{BF} + \vec{FC} = \vec{BC} = \vec{c} - \vec{a}$

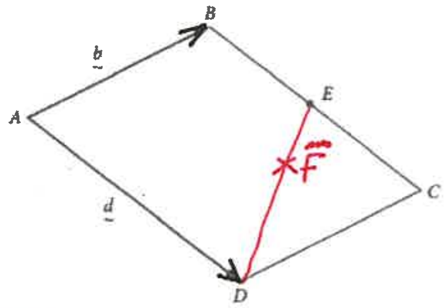
Further $\vec{FC} = 2\vec{BF}$ so $3\vec{BF} = \vec{c} - \vec{a}$ or $\vec{BF} = \frac{1}{3}(\vec{c} - \vec{a})$

So $\vec{FC} = \frac{2}{3}(\vec{c} - \vec{a})$

e) $\vec{AF} = \vec{AB} + \vec{BF} = \vec{a} + \frac{1}{3}(\vec{c} - \vec{a}) = \frac{1}{3}\vec{c} + \frac{2}{3}\vec{a}$.

$\vec{AF} = \frac{1}{3}(2\vec{a} + \vec{c})$

12 $ABCD$ is a parallelogram in which $\overrightarrow{AB} = \underset{\sim}{b}$ and $\overrightarrow{AD} = \underset{\sim}{d}$ and E is the midpoint of $\overrightarrow{BC}$.



(a) Express $\overrightarrow{AC}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$.

(b) Express $\overrightarrow{AE}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$.

(c) Express $\overrightarrow{DE}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$.

(d) If F is a point on $\overrightarrow{DE}$ and $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DE}$, express $\overrightarrow{DF}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$.

(e) Find $\overrightarrow{AF}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$, and hence show that F lies on $\overrightarrow{AC}$.

(f) Find the ratio $\overline{AF} : \overline{FC}$.

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{b} + \overrightarrow{AD} = \vec{b} + \vec{d}$

b) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \vec{b} + \frac{1}{2}\overrightarrow{BC} = \vec{b} + \frac{1}{2}\overrightarrow{AD} = \vec{b} + \frac{1}{2}\vec{d}$

c) $\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE} = \overrightarrow{AB} + (-\overrightarrow{BE}) = \vec{b} - \frac{1}{2}\vec{d}$

d) $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DE} = \frac{2}{3}\left[\vec{b} - \frac{1}{2}\vec{d}\right] = \frac{2}{3}\vec{b} - \frac{1}{3}\vec{d}$

e) $\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF} = \vec{d} + \left[\frac{2}{3}\vec{b} - \frac{1}{3}\vec{d}\right] = \frac{2}{3}\vec{b} + \frac{2}{3}\vec{d}$

So $\overrightarrow{AF} = \frac{2}{3}\left(\vec{b} + \vec{d}\right) = \frac{2}{3}\overrightarrow{AC}$

The two vectors are parallel, and share one point in common, $\therefore$ F lies on the line (AC)

f) $AF : FC$

$\overrightarrow{AC} = \overrightarrow{AF} + \overrightarrow{FC}$ so $\overrightarrow{FC} = \overrightarrow{AC} - \overrightarrow{AF}$

$\overrightarrow{FC} = \frac{3}{2}\overrightarrow{AF} - \overrightarrow{AF} = \frac{1}{2}\overrightarrow{AF}$

So the ratio $AF : FC$ is $2 : 1$