PIGEONHOLE PRINCIPLE

1 How many cards must be selected from a standard pack of 52 playing cards to make sure that you have two cards of the same suit?

There are 4 suits, so 4 pigeonholes, so we need to pick at least 5 cards.

2 If you select five cards from a standard pack of 52 playing cards, at least how many must be of the same suit?

Each of the 5 cards can belong to one of the 4 suits. So at least 2 cards must belong to the same suit.

3 There are three pairs of socks in a drawer, coloured brown, grey and black. They are not paired up. How many socks must be selected from the drawer to be sure that you have a pair of the same colour?

3 colours (3 pigeonholes). So we need to pick 4 socks to be sure we have a pair of the same colour.

4 Six pairs of shoes, of different colours and styles, have been thrown into the bottom of my wardrobe. It is dark. What is the minimum number of shoes I need to take from the wardrobe to be sure I have a matching pair?

I need to take 7 shoes from the wardrobe

5 Twenty-five students attend a class reunion and shake hands with each other. If no students shakes hands with the same person twice, explain why two students will each have shaken the same number of hands.

Each student shakes hands with 24 other students. (i.e. there are 24 pigeonholes for 25 students). So there must be 2 students who have shaken hands with the same number of students.

6 There are 400 students attending a Senior College. Explain why at least two of them will celebrate their birthday on the same day.

The year last 365 or 366 days. So there must be at least 2 persons who share a birthday.

7 (a) When taking pairs of numbers from the integers 1 to 8, list all the pairs of integers that add up to 9.
  (b) How many numbers do you need to select from the integers 1 to 8 to be sure that you have a pair of numbers that add up to 9?

a) 1 and 8, 2 and 7, 3 and 6, 4 and 5, 5/4, 6/3, 7/2, 8/1

b) So there are 4 pairs of integers that add up to 9.
There are 4 pigeonholes, so you need to pick at least 5 numbers to be sure of having 2 numbers that add up to 9

PIGEONHOLE PRINCIPLE

8 Prove that in a group of 13 people there are two people who were born in the same month.

12 months correspond to 12 pigeonholes

So, as the group is made of 13 people, there must be at least 1 month with 2 people in it, by the pigeon hole principle.

HSC - Extension - 2020 sample

7 Each of the students in an athletics team is randomly allocated their own locker from a row of 100 lockers.
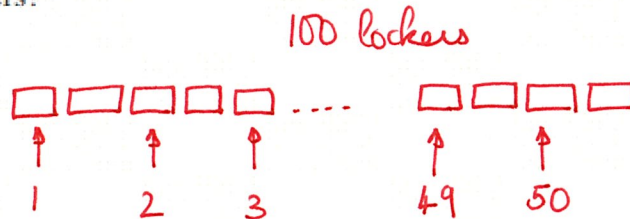
What is the smallest number of students in the team that guarantees that two students are allocated consecutive lockers?

A. 26

B. 34

C. 50

D. 51

100 lockers

HSC - Extension 1 - 2021

10 The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only.

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A. 236

B. 237

C. 238

D. 239

$$\frac{3543}{15} = 236.2$$

So $236 \times 15 = 3540$, so there must be several with 237.

So, to be elected, the smallest number of votes the new president could have received is 238 (as she needs more votes than the others)