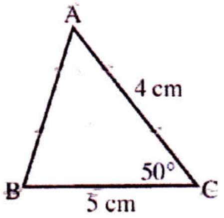
$$A = \frac{1}{2} ba \sin C$$

QUESTION 1 Find the area of each of the following triangles correct to one decimal place.

a

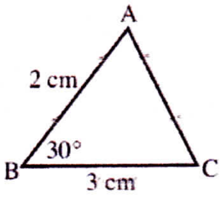


$$\text{Area} = \frac{1}{2} \times 4 \times 5 \times \sin 50$$

$$\text{Area} = \cancel{}\ cm^2$$

$$7.7$$

b

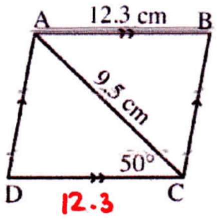


$$\text{Area} = \frac{1}{2} \times 3 \times 2 \sin 30$$

$$\text{Area} = 1.5\ cm^2$$

QUESTION 3

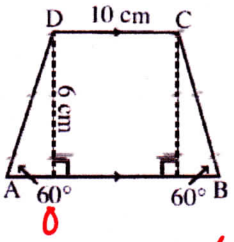
a Find the area of the parallelogram ABCD.



$$\text{Area} = 2 \times \left(\frac{1}{2} \times 12.3 \times 9.5 \times \sin 50\right)$$

$$\text{Area} = 89.5\ cm^2$$

b Find the area of the trapezium ABCD.

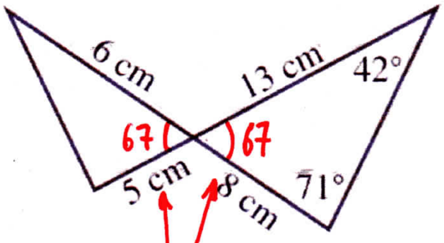


$$\text{Area} = 2 \times \left(\frac{AO \times 6}{2}\right) + 10 \times 6$$

For AO: $\tan 60 = \frac{6}{AO}$ so $AO = \frac{6}{\tan 60}$

$$\text{Area} = \frac{6 \times 6}{\tan 60} + 60$$

$$= 80.8\ cm^2$$



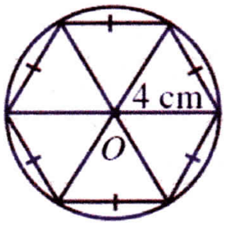
vertically opposite

Find the total area of this figure, correct to the nearest cm^2.

$$\text{Area} = \frac{1}{2} \times 8 \times 13 \sin 67 + \frac{1}{2} \times 5 \times 6 \times \sin 67$$

$$\text{Area} = \sin 67 \times (52 + 15)$$

So $\text{Area} = 62\ cm^2$



A regular hexagon has been inscribed in a circle with centre O and radius 4 cm. Find the area of the hexagon, without the use of a calculator.

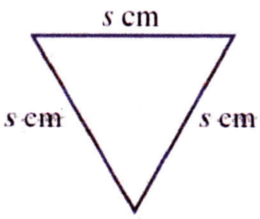
The hexagon is made of 6 triangles, so each angle is $\frac{360}{6} = 60°$

The Area of each triangle is $\frac{1}{2} \times 4 \times 4 \times \sin 60 = 8 \sin 60$

As the exact value of $\sin 60$ is $\sqrt{3}/2$, the area of each triangle is $8 \times \sin 60 = 8\frac{\sqrt{3}}{2} = 4\sqrt{3}$

There are 6 triangles, so the total area is $6 \times 4\sqrt{3} = 24\sqrt{3}\,\text{cm}^2$

Show, by trigonometry, that the area of an equilateral triangle of side s cm is given by the formula $A = \frac{\sqrt{3}}{4}s^2$.



This is an equilateral triangle, so each interior angle is 60°

$\text{Area} = \frac{1}{2} \times s \times s \times \sin 60$

But $\sin 60 = \frac{\sqrt{3}}{2}$

So $\text{Area} = \frac{1}{2}s^2 \times \frac{\sqrt{3}}{2}$ which is $\frac{\sqrt{3}}{4}s^2$

Prove the identities:

1 = $\csc\theta \ \sec\theta = \frac{1}{\sin\theta\cos\theta}$

$1 = \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

So $1 = \frac{1}{\sin\theta\cos\theta}$ ✓

A = $\csc\theta \ \tan\theta = \frac{1}{\cos\theta}$

$A = \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$

So $A = \frac{1}{\cos\theta}$ ✓

B = $\sec\theta + \csc\theta = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

$B = \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

So $B = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$ ✓

C = $\sec\theta - \cos\theta = \frac{\sin^2\theta}{\cos\theta}$

$C = \frac{1}{\cos\theta} - \cos\theta$

$C = \frac{1 - \cos^2\theta}{\cos\theta}$

But $\sin^2\theta + \cos^2\theta = 1$

so $1 - \cos^2\theta = \sin^2\theta$

So $C = \frac{\sin^2\theta}{\cos\theta}$

D = $(1 - \sin\theta)(1 + \sin\theta) = (\cos\theta)^2$

We know that:

$(a-b)(a+b) = a^2 - b^2$, so

$D = (1^2 - \sin^2\theta) = 1 - \sin^2\theta$

But $\sin^2\theta + \cos^2\theta = 1$

so $1 - \sin^2\theta = \cos^2\theta$

So $D = \cos^2\theta$ ✓

E $\frac{1+\cot\theta}{1+\tan\theta} = \cot\theta$

$E = \frac{1 + 1/\tan\theta}{1 + \tan\theta} = \frac{\frac{1}{\tan\theta}\cancel{(\tan\theta + 1)}}{\cancel{1 + \tan\theta}}$

So $E = \frac{1}{\tan\theta}$

which is $\cot\theta$.

$E = \cot\theta$ ✓

Prove the identities:

F $= (1 + \tan^2\theta)\cos^2\theta = 1$

$$F = \left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$$

$$F = \cos^2\theta + \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$$

So $F = \cos^2\theta + \sin^2\theta$

which gives: $F = 1$

G $= \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

We know $\sin^2\theta + \cos^2\theta = 1$

so $\cos^2\theta = 1 - \sin^2\theta$

So $G = 1 - \sin^2\theta - \sin^2\theta$

$G = 1 - 2\sin^2\theta$

H $= \tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta = 1$

$$H = \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cancel{\cos^2\theta} + \frac{\cos^2\theta}{\cancel{\sin^2\theta}} \times \cancel{\sin^2\theta}$$

$H = \sin^2\theta + \cos^2\theta$

So $H = 1$

I $= 3\cos^2\theta - 2 = 1 - 3\sin^2\theta$

We know $\sin^2\theta + \cos^2\theta = 1$

so $\cos^2\theta = 1 - \sin^2\theta$

$I = 3(1 - \sin^2\theta) - 2$

$I = 3 - 3\sin^2\theta - 2$

So $I = 1 - 3\sin^2\theta$

J $= \dfrac{1}{\sec\varphi - \tan\varphi} - \dfrac{1}{\sec\varphi + \tan\varphi} = 2\tan\varphi$

$$J = \frac{(\sec\varphi + \tan\varphi) - (\sec\varphi - \tan\varphi)}{(\sec\varphi - \tan\varphi)(\sec\varphi + \tan\varphi)}$$

$$J = \frac{2\tan\varphi}{\sec^2\varphi - \tan^2\varphi}$$

But: $\sec^2\varphi - \tan^2\varphi = \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\varphi}{\cos^2\varphi}$

So $\sec^2\varphi - \tan^2\varphi = \dfrac{1 - \sin^2\varphi}{\cos^2\varphi}$

$\sec^2\varphi - \tan^2\varphi = \dfrac{\cos^2\varphi}{\cos^2\varphi} = 1$

So $J = 2\tan\varphi$

K $\dfrac{1}{1 + \sin\varphi} + \dfrac{1}{1 - \sin\varphi} = 2\sec^2\varphi$

$$K = \frac{(1 - \sin\varphi) + (1 + \sin\varphi)}{(1 + \sin\varphi)(1 - \sin\varphi)}$$

$$K = \frac{2}{1 - \sin^2\varphi}$$

$$K = \frac{2}{\cos^2\varphi}$$

So $K = 2\sec^2\varphi$